Math 1152, Quiz #5, Spring 2014 Name KEY

Instructions: Show all work. Use *exact* answers unless specifically asked to round. You may check your answers in the calculator, but you must show work to receive credit.

1. For each of the improper integrals below, determine whether the integral converges or diverges. If it converges, determine what it converges to.

   a. $\int_0^2 \frac{dx}{\sqrt{4-x^2}}$

$$\lim_{b\to 2} \int_0^b \frac{dx}{\sqrt{4-x^2}} = \lim_{b\to 2} \arcsin\left(\frac{x}{2}\right)\Big|_0^b =$$

$$\lim_{b\to 2} \arcsin\left(\frac{b}{2}\right) - \cancel{\arcsin(0)} = \arcsin 1 = \pi/2$$

$$\boxed{\text{Converges}}$$

   b. $\int_{-\infty}^{\infty} \frac{x\,dx}{1+x^2}$

$$\lim_{a\to -\infty} \int_a^0 \frac{x}{1+x^2}\,dx + \lim_{b\to\infty} \int_0^b \frac{x}{1+x^2}\,dx$$

$u = 1+x^2$

$du = 2x\,dx \Rightarrow \frac{1}{2}du = x\,dx$

$\int \frac{x}{1+x^2}dx = \int \frac{1}{2}du \cdot \frac{1}{u}$

$$\lim_{a\to -\infty} \frac{1}{2}\ln|1+x^2|\Big|_a^0 + \lim_{b\to\infty} \frac{1}{2}\ln|1+x^2|\Big|_0^b$$

$$\lim_{a\to -\infty} \frac{1}{2}\ln(1) - \frac{1}{2}\ln(1+a^2) = -\infty \quad \cdot \lim_{b\to\infty} \frac{1}{2}\ln|1+b^2| - \frac{1}{2}\ln(1) = \infty$$

$$\boxed{\text{diverges}}$$

2. Find the derivative of $f(x) = \tanh(x^2) + \cosh^2(x)$.

$$2x\,\text{sech}^2(x^2) + 2\cosh x \sinh x$$

3. Integrate $\int \tanh(x)\,dx$. [Hint: Write as $\frac{\sinh(x)}{\cosh(x)}$.]

$u = \cosh x$

$du = \sinh x$

$$\int \tanh x\,dx = \int \frac{\sinh x}{\cosh x}dx = \int \frac{1}{u}du = \ln|u| \Rightarrow$$

$$\ln|\cosh x| + C$$

or $\ln(\cosh x) + C$ since $\cosh x > 0$