

Instructions: Show all work. Use exact answers unless specifically asked to round. You may check your answers in the calculator, but you must show work to receive credit.

1. Integrate.

a. $\int e^x \sec^4(e^x) dx$ $u = e^x \quad du = e^x \Rightarrow \int \sec^4 u \, du$

$$\int \sec^2 u (1 + \tan^2 u) \, du = \int \sec^2 u + \tan^2 u \sec^2 u \, du$$

$$= \tan u + \frac{1}{3} \tan^3 u + C$$

$$\boxed{= \tan e^x + \frac{1}{3} \tan^3 e^x + C}$$

b. $\int \sin^2 x \cos^2 x \, dx$ $\frac{1}{4} \int (1 - \cos 2x)(1 + \cos 2x) \, dx = \frac{1}{4} \int 1 - \cos^2 2x \, dx$

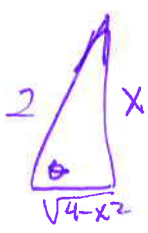
$$= \frac{1}{4} \int \frac{1}{2} - \frac{1}{2} \cos 4x \, dx = \frac{1}{8} \left[x - \frac{1}{4} \sin 4x \right] + C$$

$1 - \frac{1}{2}(1 + \cos 4x)$

c. $\int \frac{dx}{x^2 \sqrt{4-x^2}}$ $x = 2 \sin \theta$ $dx = 2 \cos \theta \, d\theta$ $\sqrt{4-x^2} = 2 \cos \theta$

$$\int \frac{2 \cos \theta}{(2 \sin \theta)^2 \cdot 2 \cos \theta} \, d\theta$$

$$= \frac{1}{4} \int \csc^2 \theta \, d\theta = -\frac{1}{4} \cot \theta + C$$

$$\boxed{= -\frac{1}{4} \left(\frac{\sqrt{4-x^2}}{x} \right) + C}$$


d. $\int \frac{(1+x^2)^{3/2} dx}{x^6}$ $x = \tan \theta$ $dx = \sec^2 \theta \, d\theta$ $(1 + \tan^2 \theta)^{3/2} = (\sec^2 \theta)^{3/2}$

$$= \int \frac{\sec^3 \theta \cdot \sec^2 \theta}{\tan^6 \theta} \, d\theta = \int \frac{\cos^6 \theta}{\sin^6 \theta} \cdot \frac{1}{\cos^5 \theta} \, d\theta = \int \frac{\cos \theta}{\sin^6 \theta} \, d\theta =$$

$$u = \sin \theta \quad du = \cos \theta \quad \int u^{-6} \, du = -\frac{1}{5} u^{-5} = -\frac{1}{5} \cos^5 \theta + C = \boxed{-\frac{1}{5} \left(\frac{\sqrt{1+x^2}}{x} \right)^5 + C}$$
