Math 1152, Quiz #13, Spring 2014 Name KEY

Instructions: Show all work. Use *exact* answers unless specifically asked to round. You may check your answers in the calculator, but you must show work to receive credit.

1. Find the Maclaurin polynomial P_4 for $f(x) = e^{2x}$. You may use the table below to aid your calculations. $c=0$

n	n!	$f^{(n)}(x)$	$f^{(n)}(c)$	$(x-c)^n$	$\frac{f^{(n)}(c)}{n!}(x-c)^n$
0	1	e^{2x}	1	$x^0=1$	$\frac{1}{1}(1) = 1$
1	1	$2e^{2x}$	2	x	$\frac{2}{1}x = 2x$
2	2	$4e^{2x}$	4	x^2	$\frac{4}{2}x^2 = 2x^2$
3	6	$8e^{2x}$	8	x^3	$\frac{8}{6}x^3 = \frac{4}{3}x^3$
4	24	$16e^{2x}$	16	x^4	$\frac{16}{24}x^4 = \frac{2}{3}x^4$

$$P_4(x) = 1 + 2x + 2x^2 + \frac{4}{3}x^3 + \frac{2}{3}x^4$$

2. Find the Taylor polynomial of degree three that approximates the function $f(x) = \sqrt{4+x}$ centered at c=2.

n	n!	$f^{(n)}(x)$	$f^{(n)}(c)$	$(x-c)^n$	$\frac{f^{(n)}(c)}{n!}(x-c)^n$
0	1	$(4+x)^{1/2}$	$\sqrt{6}$	$(x-2)^0=1$	$\sqrt{6}$
1	1	$\frac{1}{2}(4+x)^{-1/2}$	$\frac{1}{2\sqrt{6}}$	$x-2$	$\frac{1}{2\sqrt{6}}(x-2)$
2	2	$-\frac{1}{4}(4+x)^{-3/2}$	$\frac{-1}{24\sqrt{6}}$	$(x-2)^2$	$\frac{-1}{24\sqrt{6}}(x-2)^2$
3	6	$\frac{3}{8}(4+x)^{-5/2}$	$\frac{1}{96\sqrt{6}}$	$(x-2)^3$	$\frac{1}{576\sqrt{6}}(x-2)^3$
4	24	$-\frac{15}{16}(4+x)^{-7/2}$	$\frac{-5}{27648\sqrt{6}}$	$(x-2)^4$	$\frac{-5}{1152}(x-2)^4$

$$P_3(x) = \sqrt{6} + \frac{1}{2\sqrt{6}}(x-2) - \frac{1}{24\sqrt{6}}(x-2)^2 + \frac{1}{576\sqrt{6}}(x-2)^3$$

max value of this function is at $x=1$ on this interval

How large is the error on the interval [1,3]?

$$R_3 \le \frac{-15}{16\cdot 24\cdot 5^{7/2}}(1)^4 = \left|\frac{-1}{3200\sqrt{5}}\right| \approx 1.4\times10^{-4}$$