Math 1152, Quiz #10, Spring 2014 Name KEY

Instructions: Show all work. Use *exact* answers unless specifically asked to round. You may check your answers in the calculator, but you must show work to receive credit.

1. Find $\langle 1,-2,5\rangle \times \langle -3,7,4\rangle$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 5 \\ -3 & 7 & 4 \end{vmatrix} = (-8-35)\hat{i} - (4+15)\hat{j} + (7-6)\hat{k}$$

$$= -43\hat{i} - 19\hat{j} + \hat{k}$$

$$\langle -43, -19, 1\rangle$$

2. What is the length of the curve given by $x = 2t - 1, y = t^{3/2}$ on the interval [0,1].

$$\frac{dx}{dt} = 2 \qquad \frac{dy}{dt} = \frac{3}{2}t^{1/2}$$

$$S = \int_0^1 \sqrt{4 + \frac{9}{4}t}\, dt$$

$u = 4 + \frac{9}{4}t$ $\quad t=1 \Rightarrow u = \frac{25}{4}$

$du = \frac{9}{4}dt$ $\quad t=0 \Rightarrow u = 4$

$\frac{4}{9}du = dt$

$$= \int_4^{25/4} u^{1/2} \cdot \frac{4}{9}\, du = \frac{4}{9}\cdot\frac{2}{3}u^{3/2}\Big|_4^{25/4} = \frac{8}{27}\left[\frac{125}{8} - 8\right] = \frac{61}{27}$$

3. Find an expression for the nth term in the sequence $0, \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \ldots$

$0, \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \ldots$

$\uparrow$ $n=1$

$a_n = \frac{n-1}{n}$ $\quad n=1$ $\qquad$ or $\qquad$ $a_n = \frac{n}{n+1}$ $\quad n=0$

4. Find the limit of the sequence $a_n = \frac{\sin n}{n}$, if it exists, or state that the sequence diverges.

$\lim_{n\to\infty} \frac{\sin n}{n} = 0$ by Squeeze Theorem

$-1 \le \sin n \le 1 \Rightarrow -\frac{1}{n} \le \frac{\sin n}{n} \le \frac{1}{n}$

$\lim_{n\to\infty} -\frac{1}{n} = 0$ & $\lim_{n\to\infty} \frac{1}{n} = 0$ so $0 \le \lim_{n\to\infty} \frac{\sin n}{n} \le 0$