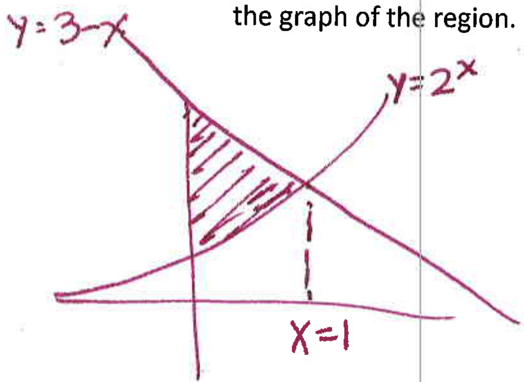
Math 1152, Exam #1, Spring 2014 Name KEY

Instructions: Show all work. Use exact answers unless specifically asked to round. You may check your answers in the calculator, but you must show work to get full credit. Incorrect answers with no work will receive no credit. Be sure to complete all the requested elements of each problem.

1. Find the area of the region bounded by the graphs of $y = 2^x, y = 3 - x$ and the y-axis. Sketch the graph of the region. (10 points)

$x=0$



$$\int_0^1 3-x-2^x\,dx =$$

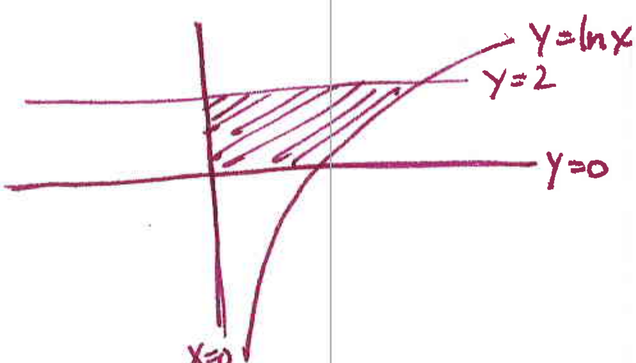
$$3x-\frac{1}{2}x^2-\frac{2^x}{\ln 2}\Big|_0^1 =$$

$$3-\frac{1}{2}-\frac{2}{\ln 2}-\left(0-0-\frac{1}{\ln 2}\right) =$$

$$\boxed{\frac{5}{2}-\frac{1}{\ln 2}} \approx 1.057304959$$

2. Find the area of the region (using horizontal slices) bounded by $y = 2, y = \ln x, x = 0, y = 0$. Sketch the region. (10 points)

left



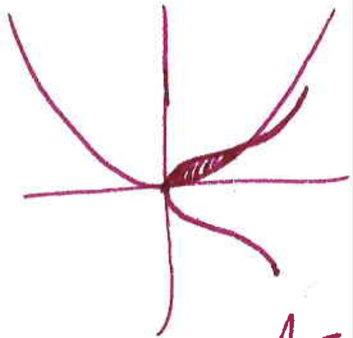
$y=\ln x \Rightarrow x=e^y$
right

$$\int_0^2 e^y - 0\,dy$$

$$= e^y\Big|_0^2 = \boxed{e^2-1}$$

≈ 6.389056099

3. Set up, but do not integrate, the integral to find the area of the region bounded by $y = x^2$ and $x = 2sin^2y$. (6 points)



$\frac{x}{2} = \sin^2 y \quad \pm\sqrt{\frac{x}{2}} = \sin y \quad y = \pm \arcsin\sqrt{\frac{x}{2}}$

$\text{top} = \arcsin\sqrt{\frac{x}{2}}$

$\text{bottom} = x^2$

intersects at 0 and approx. .83953353

call it b

$$A = \int_0^b \sin^{-1}\sqrt{\frac{x}{2}} - x^2 \, dx$$

4. Find the length of the arc on the curve $y = \sqrt{x-2}$ on the interval [3,4]. Set up the integral, and then you may use your calculator to evaluate it numerically. (7 points)

$$y' = \frac{1}{2}(x-2)^{-1/2} = \frac{1}{2\sqrt{x-2}}$$

$$s = \int_3^4 \sqrt{1 + \left(\frac{1}{2\sqrt{x-2}}\right)^2}\, dx = \int_3^4 \sqrt{1 + \frac{1}{4(x-2)}}\, dx = \int_3^4 \sqrt{\frac{4x-8+1}{4x-8}}\, dx$$

$$= \int_3^4 \sqrt{\frac{4x-7}{4x-8}}\, dx \approx 1.08306428$$

5. Find the derivative of $y = \tanh^2 x$. Use this information to find the slope of the tangent line at x=ln(2). [Hint: once you find the derivative, you may find it helpful to use the definition of the hyperbolic trig functions to simplify the expressions with the x-value plugged in.] (10 points)

$$y' = 2\tanh x \,\mathrm{sech}^2 x$$

$$\tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}} \qquad \mathrm{sech}\, x = \frac{2}{e^x + e^{-x}}$$

$$= 2\left(\frac{3}{5}\right)\left(\frac{4}{5}\right)^2 = 2\left(\frac{3}{5}\right)\left(\frac{16}{25}\right)$$

$$= \frac{96}{125}$$

$$\tanh(\ln 2) = \frac{e^{\ln 2} - e^{-\ln 2}}{e^{\ln 2} + e^{-\ln 2}} = \frac{2 - 1/2}{2 + 1/2} = \frac{3/2}{5/2} = \frac{3}{5}$$

$$\mathrm{sech}(\ln 2) = \frac{2}{e^{\ln 2} + e^{-\ln 2}} = \frac{2}{2 + 1/2} = \frac{2}{5/2} = 2 \cdot \frac{2}{5} = \frac{4}{5}$$

tangent line

$$\boxed{y - \frac{9}{25} = \frac{96}{125}(x - \ln 2)}$$

$$y = \tanh^2(\ln 2) = \left(\frac{3}{5}\right)^2 = \frac{9}{25}$$

6. Integrate. Use any technique which is appropriate. (7 points each)

a. $\int \cosh^3 x \sinh x \, dx$

$du = \sinh x \, dx \quad u = \cosh x$

$\int u^3 \, du$

$\frac{u^4}{4} + C = \frac{1}{4}\cosh^4 x + C$

$u = x^2+4$

$du = 2x\,dx \Rightarrow \frac{1}{2}du = x\,dx$

b. $\int \frac{x+2}{x^2+4} dx = \int \frac{x}{x^2+4} dx + \int \frac{2}{x^2+4} dx =$

$a = 2$

$\frac{1}{2}\ln|x^2+4| + \frac{2}{2}\arctan\left(\frac{x}{2}\right) + C$

c. $\int x \sec^2 x \, dx$

$u = x \qquad dv = \sec^2 x \, dx$

$du = dx \qquad v = \tan x$

$x\tan x - \int \tan x \, dx =$

$x\tan x + \ln|\cos x| + C$

d. $\int 6\csc^4 x\,dx$ $= 6\int \csc^2 x\,(1+\cot^2 x)\,dx$

$u = \cot x$

$-du = \csc^2 x\,dx$

$-6\int 1+u^2\,du = -6\left[u + \frac{u^3}{3}\right] + C$

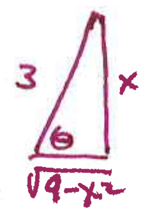
$-6\cot x - 2\cot^3 x + C$

e. $\int \frac{\sqrt{9-x^2}}{x^2}\,dx$ $x = 3\sin\theta$ $dx = 3\cos\theta\,d\theta$ $\sqrt{9-x^2} = 3\cos\theta$

$\int \frac{\cancel{3}\cos\theta\,d\theta \cdot \cancel{3}\cos\theta}{\cancel{9}\sin^2\theta} = \int \frac{\cos^2\theta}{\sin^2\theta}\,d\theta = \int \cot^2\theta\,d\theta =$

$\int \csc^2\theta - 1\,d\theta = -\cot\theta - \theta + C$

$\frac{x}{3} = \sin\theta$



$= -\frac{\sqrt{9-x^2}}{x} - \sin^{-1}\left(\frac{x}{3}\right) + C$

f. $\int \frac{2}{x^3+x^2}\,dx$ $= \int \frac{2}{x^2(x+1)}\,dx = \int \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+1}\,dx$

$2 = Ax(x+1) + B(x+1) + Cx^2$

$x=0$ $2 = B(1) \Rightarrow B = 2$

$x=-1$ $2 = C(1) \Rightarrow C = 2$

$x=1$ $2 = A(1)(2) + 2(2) + 2(1)$

$2 = 2A + 6 \Rightarrow -4 = 2A \Rightarrow A = -2$

$= \int \frac{-2}{x} + \frac{2}{x^2} + \frac{2}{x+1}\,dx$

$-2\ln|x| - \frac{2}{x} + 2\ln|x+1| + C$

or $2\ln\left|\frac{x+1}{x}\right| - \frac{2}{x} + C$

or $\ln\left(\frac{x+1}{x}\right)^2 - \frac{2}{x} + C$

7. Set up the partial fraction decomposition of the expression $\frac{2x^3+8}{x(x+1)^3(x-3)(x^2+4)}$. Do not solve for any of the constants. (5 points)

$$\frac{A}{X} + \frac{B}{X+1} + \frac{C}{(X+1)^2} + \frac{D}{(X+1)^3} + \frac{E}{X-3} + \frac{Fx+G}{x^2+4}$$

8. Evaluate the following improper integrals and determine if they converge or diverge. If they converge, say what they converge to. (8 points each)

a. $\int_0^{\infty} xe^{-x^2}dx$

$u = -x^2$

$du = -2x\,dx \Rightarrow -\frac{1}{2}du = x\,dx$

when $x=0 \Rightarrow u=0$ when $x \to \infty,\ u \to -\infty$

$$\int_0^{-\infty} -\frac{1}{2}e^u\,du = \lim_{a\to-\infty}\int_a^0 \frac{1}{2}e^u\,du = \lim_{a\to-\infty}\frac{1}{2}e^u\Big|_a^0 =$$

$$\lim_{a\to-\infty}\frac{1}{2}e^0 - \cancel{\frac{1}{2}e^{a}}^{\,a\to 0} = \frac{1}{2} - 0 = \boxed{\frac{1}{2}}$$ converges

b. $\int_{-2}^{2}\frac{dx}{\sqrt{4-x^2}}$

$$\lim_{a\to-2}\int_a^0 \frac{dx}{\sqrt{4-x^2}} + \lim_{b\to 2}\int_0^b \frac{dx}{\sqrt{4-x^2}} =$$

$$\lim_{a\to-2} \arcsin\left(\frac{x}{2}\right)\Big|_a^0 + \lim_{b\to2}\arcsin\left(\frac{x}{2}\right)\Big|_0^b =$$

$$\lim_{a\to-2}\sin^{-1}(0) - \sin^{-1}\left(\frac{a}{2}\right) + \lim_{b\to2}\sin^{-1}\left(\frac{b}{2}\right) - \sin^{-1}(0) =$$ Converges

$$-\sin^{-1}(-1) + \sin^{-1}(1) = -\left(-\frac{\pi}{2}\right) + \frac{\pi}{2} = \boxed{\pi}$$

9. Use integration by tables to find $\int (2x-1)^4 \cos(5x)\, dx$ (10 points)

tabular method

+/–	u	dv
+	$(2x-1)^4$	$\cos(5x)$
–	$8(2x-1)^3$	$\frac{1}{5}\sin(5x)$
+	$48(2x-1)^2$	$-\frac{1}{25}\cos(5x)$
–	$192(2x-1)$	$-\frac{1}{125}\sin(5x)$
+	384	$\frac{1}{625}\cos(5x)$
–	0	$\frac{1}{3125}\sin(5x)$

$$\frac{1}{5}(2x-1)^4\sin(5x) + \frac{8}{25}(2x-1)^3\cos(5x)$$
$$-\frac{48}{125}(2x-1)^2\sin(5x) -$$
$$\frac{192}{625}(2x-1)\cos(5x) +$$
$$\frac{384}{3125}\sin(5x) + C$$

10. Set up with a valid substitution, but do not attempt to integrate $\int tan^{17}x\, sec^{12}x dx$. (9 points)

version 1: $\int \tan^{17}x \sec^{10}x \sec^2 x\, dx =$

$\int \tan^{17}x (1+\tan^2 x)^5 \sec^2 x\, dx$ $\quad u = \tan x$, $du = \sec^2 x\, dx$

$\int u^{17}(1+u^2)^5\, du$

version 2: $\int \tan^{16}x \sec^{11}x\, (\sec x \tan x)\, dx$

$\int (\sec^2 x - 1)^8 \sec^{11}x\, (\sec x \tan x)\, dx$

$u = \sec x$

$du = \sec x \tan x\, dx$

$\int (u^2-1)^8 u^{11}\, du$

11. Use the definition of $\sinh(x) = \frac{e^x - e^{-x}}{2}$ to prove that $\int \sinh x \, dx = \cosh x + C$. (4 points)

$$\int \sinh x \, dx = \int \frac{e^x - e^{-x}}{2} dx =$$

$$\frac{1}{2} \int e^x - e^{-x} dx = \frac{1}{2}\left[e^x + e^{-x}\right] + C$$

$$= \cosh x + C$$

Math 1152, Exam #1, Integration by Tables Problem, Spring 2014 Name KEY

Instructions: Use the Integrations Tables attached to complete the integration problems on this page. Note, substitution may be required to match the function up with something in the table. State which formula you use. (7 points each)

1. $\int \frac{5}{3x^2(2x-7)^2}\,dx = \frac{5}{3}\int \frac{1}{x^2(2x-7)^2}\,dx$ formula (2.20)

$$\int \frac{1}{x^2(ax+b)^2}\,dx = \frac{2a}{b^3}\ln\left|\frac{ax+b}{x}\right| - \frac{2ax+b}{b^2x(ax+b)} + C$$

$$\frac{5}{3}\left[-\frac{4}{343}\ln\left|\frac{2x-7}{x}\right| - \frac{4x-7}{49x(2x-7)}\right] + C$$

2. $\int 4e^{6x}\arctan(e^{3x}+1)\,dx$ [Hint: Write e^{6x} as $e^{3x}\cdot e^{3x}$.]

$$4\int e^{3x}\cdot e^{3x}\arctan(e^{3x}+1)\,dx$$

$u = e^{3x}+1$
$du = 3e^{3x}\,dx$
$\frac{1}{3}du = e^{3x}\,dx$
$u-1 = e^{3x}$

$$\frac{4}{3}\int (u-1)\arctan u\,du$$

$$= \frac{4}{3}\left[\int u\arctan u\,du - \int \arctan u\,du\right]$$

(5.7) (5.6)

$\frac{1}{2}x^2\arctan x + \frac{1}{2}\arctan x - \frac{1}{2}x + C$ $\Rightarrow$ $x\arctan x - \ln\sqrt{1+x^2} + C$

$$\frac{4}{3}\left[\frac{1}{2}u^2\arctan u + \frac{1}{2}\arctan u - \frac{1}{2}u + u\arctan u - \ln\sqrt{1+u^2}\right] + C$$

$$\frac{4}{3}\left[\frac{1}{2}(e^{3x}+1)^2\arctan(e^{3x}+1) + \frac{1}{2}\arctan(e^{3x}+1) - \frac{1}{2}(e^{3x}+1) + (e^{3x}+1)\arctan(e^{3x}+1) - \ln\sqrt{1+(e^{3x}+1)^2}\right] + C$$