Math 1050, Exam #1, Spring 2014 Name KEY

Instructions: Show all work. Use **exact** answers unless specifically asked to round. Problems with no work cannot receive partial credit. Draw diagrams to help organize the data (this is worth partial credit). If you do your work on scrap paper, you should indicate that directly on the test paper along with your final answer. It is preferable, if you can, to do work directly on the test.

1. For the numbers in the set $\left\{\frac{58}{11}, -1, 10, \sqrt{49}, 1.14, 38000, \sqrt{17}, \frac{\pi^3}{\sqrt{2}}, \frac{0}{3}, 0.\overline{27}, \frac{27}{9}\right\}$, determine which of the numbers belongs to each of the number types below. (8 points)
   a. Natural Numbers

   $\{10, \sqrt{49}, 38000, \frac{27}{9}\}$

   b. Integers

   $\{-1, 10, \sqrt{49}, 38000, \frac{0}{3}, 27/9\}$

   c. Rational Numbers

   $\{\frac{58}{11}, -1, 10, \sqrt{49}, 1.14, 38000, \frac{0}{3}, 0.\overline{27}, \frac{27}{9}\}$

   d. Irrational Numbers

   $\{\sqrt{17}, \frac{\pi^3}{\sqrt{2}}\}$

2. $4 + 3 = 3 + 4$ is an example of which property of real numbers? (2 points)

   Commutative property of addition

3. Simplify the expression. $\frac{1}{2}(8k - 12) + \frac{2}{3}(15 - 6k)$ (3 points)

   $4k - 6 + 10 - 4k = 4$

4. Evaluate the expression $\frac{4xy-2z^2}{3xz+y^3}$ for $x = 1, y = 2, z = -3$. (4 points)

   $$\frac{4(1)(2) - 2(-3)^2}{3(1)(-3) + (2)^3} = \frac{8 - 2(9)}{-9 + 8} = \frac{8 - 18}{-1} = \frac{-10}{-1} = \boxed{10}$$

5. Explain why division into zero (ex. $\frac{0}{3}$) is equal to zero, but division by zero (ex. $\frac{3}{0}$) is not defined. [Hint: division is related to a multiplication problem. Which multiplication problems are these expressions related to?] (3 points)

$\frac{0}{3} = 0$ relates to the multiplication problem $0 \times 3 = 0$ which is true. But

$\frac{3}{0} = a$ relates to $a * 0 = 3$. Since nothing * 0 can be equal to 3, a cannot exist, so not defined.

6. Simplify the following expression as much as possible. $\left(\frac{5^2-2(-13)(1)}{7\cdot 3-6^2-2}\right)^2$ (4 points)

$\left(\frac{25+26}{21-36-2}\right)^2 = \left(\frac{51}{-17}\right)^2 = (-3)^2 = 9$

7. The following problem below has at least one error in it (possibly more than one). Find the error(s), explain why they are wrong, and solve the problem correctly. (4 points)

$$\frac{1}{4}(8x-12) - \frac{1}{6}(18x-24)$$

$$12\left[\frac{1}{4}(8x-12) - \frac{1}{6}(18x-24)\right]$$
$$3(8x-12) - 2(18x-24)$$
$$24x - 36 - 36x - 48$$
$$-12x - 84$$
$$\frac{-12x}{-12} = \frac{84}{-12}$$
$$x = -7$$

① Cannot clear fractions w/o = sign

② negative sign needs to be distributed

③ Can't introduce an = sign here.

$\frac{1}{4}(8x-12) - \frac{1}{6}(18x-24)$

$= 2x - 3 - 3x + 4$

$-x + 1$

8. Solve the equation. $\frac{q}{3}+\frac{4}{5}=\frac{5q+12}{15}$ (4 points)

*15

$5q+12=5q+12$

q is all real #s since this is an identity

9. Below are three solved linear equations. For each one state the solution set, and whether the equation is an identity, a conditional equation, or a contradiction. (3 points each)

a.
$3(x+4)=2x+7(x-1)$
$3x+12=2x+7x-7$
$3x+12=9x-7$
$-6x+12=-7$
$-6x=-19$
$x=\frac{19}{6}$

$x=\left\{\frac{19}{6}\right\}$

Conditional

b.
$6(x-2)+11=4(x-1)+2x$
$6x-12+11=4x-4+2x$
$6x-1=6x-4$
$-1=-4$

$x=\emptyset$

Contradiction

10. Solve the formulas below for the specified variable. (3 points each)

a. $C = \frac{5}{9}(F - 32)$, for F.

$\frac{9}{5}C = F - 32$

$\frac{9}{5}C + 32 = F$

b. $2x - 3y = 14$, for y.

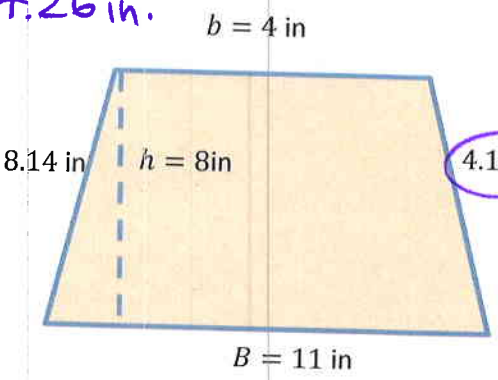
$\frac{2x - 14}{3} = y$

11. The area formula for a trapezoid is given by $A = \frac{1}{2}h(B + b)$. Find the area and perimeter of the trapezoid shown below. (4 points)

$P = 4 + 4.12 + 11 + 8.14 = 27.26$ in.

using given #'s

(should be 31.28) in.

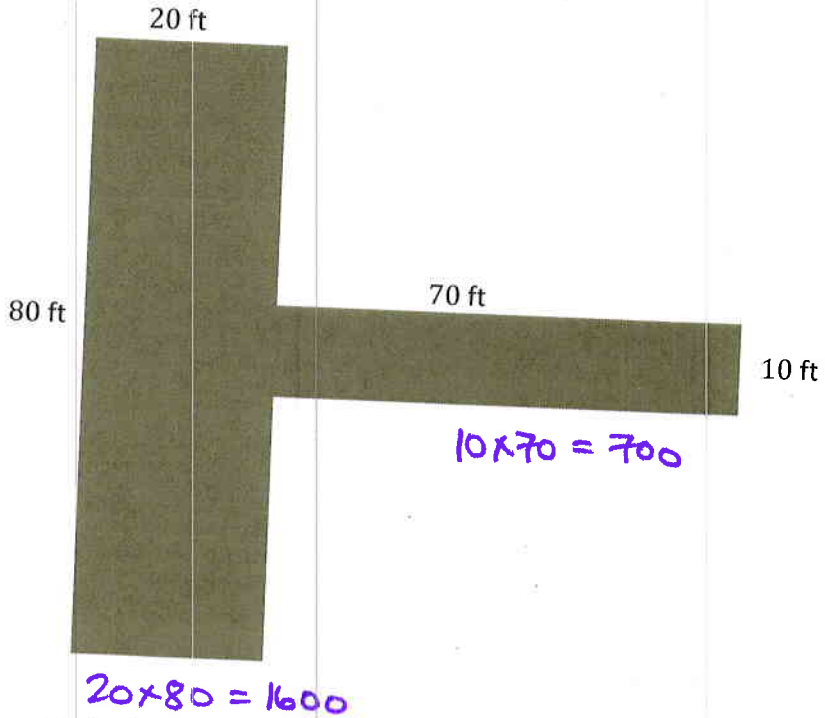


typo

= 8.14 in

$A = \frac{1}{2}(8)(4 + 11) = 60$ in^2

12. A city decides to build a new pier with a long dock, and a boardwalk area on the shore in the configuration shown below. Both sections will need to be laid down with wood planks. Find the area of the region to be constructed. (3 points)



$20 \times 80 = 1600$

$10 \times 70 = 700$

$1600 + 700 = 2300 \text{ ft}^2$

13. Translate the following statements into algebraic expressions. (Do not solve.) (2 points each)

a. The quotient of a number and 7

$\frac{x}{7}$

b. Three times the sum of a number and 4

$3(x+4)$

c. 50 less than half a number is the same as twice the number and one.

$\frac{1}{2}x - 50 = 2x + 1$

d. 49 is less than twice a number.

$49 < 2x$

14. The sum of three consecutive even integers is 72. Find all three integers. (4 points)

$n + n+2 + n+4 = 72$

$3n + 6 = 72$

$3n = 66$

$n = 22$

$n, n+2, n+4 =$

$22, 24, 26$

15. A paperback edition of a book costs \$12.50 less than the hardback edition of the book. If you purchase one of each edition, you will pay \$37.40. Find the cost of the paperback edition of the book. (4 points)

$P = h - 12.50 = \$12.45$

$h + h - 12.50 = 37.40$

$2h = 49.90$

$h = \$24.95$

16. Going into the final exam, which is worth two test grades, Brooke has test scores of 80, 83, 71, 61 and 95. What score does Brooke need to score on the final exam in order to have an average of 80? (4 points)

$$\frac{80+83+71+61+95+2x}{7} = 80$$

$390 + 2x = 560$

$\frac{2x}{2} = \frac{170}{2}$

$x = 85$

17. Find the missing values in the following percent problems: **7% of what number is 200?** (2 points)

$.07x = 200$

$x = \frac{200}{.07} \approx 2857.14$

$= \frac{20,000}{7}$

18. In a certain county, the sales tax is 6.5%. A woman purchases a used car for $7668 including tax. What was the sticker price of the car? (5 points)

$7668 = x + .065x$

$7668 = 1.065x$

$x = \$7200$

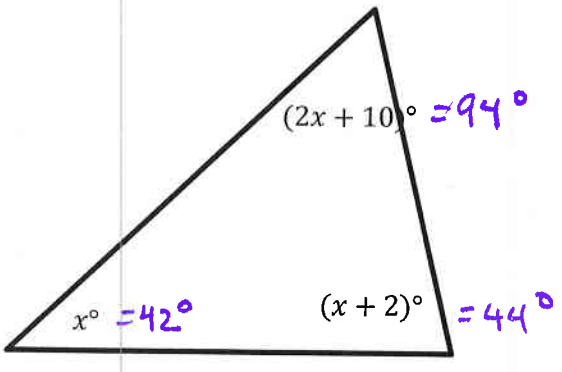
19. Find the values of each angle in the triangle below. (4 points)

$2x+10+x+2+x = 180$

$4x+12 = 180$

$4x = 168$

$x = 42$



20. Max lives on a river 30 miles from town. Max travels downstream (with the current) at 20 mph. Returning upstream (against the current), his rate is 12 mph. If the total trip to town and back took 4 hours, how long did his upstream trip take? (5 points)

down $r = 20 = b + c$

up $r = 12 = b - c$

$\frac{d}{r} = t$ $\quad \frac{30}{20} + \frac{30}{12} = 4$

upstream time is 2.5 hours.

21. If you have an angle that is 75°, give an angle that is: (2 points each)
    a. Complementary

15°

    b. Supplementary

105°

22. Explain why it's not possible for two angles in a triangle to be obtuse. (3 points)

all the angles in a triangle must add up to 180°. If one angle is obtuse this leaves less than 90° to divide between the two remaining angles. So both must be acute.

23. A sales manager received a 13% increase in salary over last year and now he makes $68,930. He's forgotten his old salary and wishes to calculate it. He sets up the problem as shown below. Explain what is wrong with his reasoning. (3 points)

$$\$68{,}930 \times 0.13 = \$8960.90$$

Therefore, his old salary was

$\$59{,}969.10$.

13% is not based on his new salary but rather his old one.

$x + .13x = 68{,}930 \Rightarrow 1.13x = 68930$

$x = \$61{,}000$

24. Solve the following inequalities. For each problem, write the answer in i) set notation, ii) on a number line, iii) in interval notation, where such expressions exist. (4 points each)

a. $3x - 1 \geq 3 + x$

$2x - 1 \geq 3$

$2x \geq 4$

$x \geq 2$

i) $\{x \mid x \geq 2\}$

ii) 

iii) $[2, \infty)$

b. $5(n+2) - 2n < 3(n+4)$

$5n + 10 - 2n < 3n + 12$

$3n + 10 < 3n + 12$

$10 < 12$

true for all x

i) $\{x \mid x \text{ is a real \#}\}$

ii) 

iii) $(-\infty, \infty)$

25. Explain how you recognize when the solution set of an inequality is all real numbers and when it has no solution (is the empty set). (2 points)

check the truth value of the final inequality if it's true, then the solution is all real #'s. (like 24b). If it's false (like $0 < 4$) then there is no solution.

26. Compare and contrast solving an inequality with solving an equation. How are the two procedures similar? How are they different? (3 points)

The procedures are largely the same, but you do have to be careful to flip the sign of the inequality when dividing by a negative, or when rearranging the equation, such as $4 > x$ is the same as $x < 4$.

Other procedures we've learn work exactly the same.