1. $100g = .1kg = m$ $5cm = .05m$

$.1 \cdot 9.8 = k \cdot .05m \Rightarrow k = 196$ $\gamma = 0$

$.1y'' + 19.6y = 0$ $\times 10 \Rightarrow$ $y'' + 196y = 0$

$r^2 + 196 = 0 \Rightarrow r = \pm 14i$ $y_c(t) = c_1 \cos 14t + c_2 \sin 14t$

$y(0) = 0$, $y'(0) = -10\ cm/sec = -.01\ m/sec$

$0 = c_1$ $y'(t) = 14c_2 \cos 14t$ $-.01 = 14c_2 \Rightarrow c_2 = \frac{-.01}{14}$

$y(t) = \frac{-.01}{14}$ $0 = y$ when $\cos 14t = 0 \Rightarrow 14t = \pi/2 \Rightarrow t = \pi/28$

2. $16\ lbs \Rightarrow \frac{16}{32} = \frac{1}{2}\ slugs = m$ $16 = k \cdot \frac{1}{4} \Rightarrow k = 64$ $\gamma = 2$

$y(0) = 0$, $y'(0) = 1/4$

$\frac{1}{2}y'' + 2y' + 64y = 0 \Rightarrow y'' + 4y' + 128y = 0$

$r^2 + 4r + 128 = 0$ $\frac{-4 \pm \sqrt{16 - 4(128)}}{2} = \frac{-4 \pm \sqrt{496}\,i}{2} = \frac{-4 \pm 4\sqrt{31}\,i}{2}$

$r = -2 \pm 2\sqrt{31}\,i$

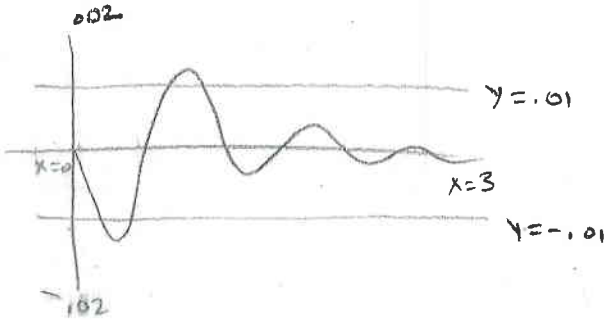
$y_c(t) = c_1 e^{-2t} \cos(2\sqrt{31}\,t) + c_2 e^{-2t} \sin(2\sqrt{31}\,t)$

$0 = c_1$

$y_c'(t) = -2c_1 e^{-2t} \sin(2\sqrt{31}\,t) + 2\sqrt{31}\,c_2 e^{-2t} \cos(2\sqrt{31}\,t)$

$-1/4 = 2\sqrt{31}\,c_2 \Rightarrow c_2 = \frac{-1}{8\sqrt{31}} = \frac{-\sqrt{31}}{248}$

$y(t) = \frac{-\sqrt{31}}{248} e^{-2t} \sin(2\sqrt{31}\,t)$



$t = .214479\ sec.$

3. $8 \text{ lbs} = \frac{8}{32} = \frac{1}{4}$ slugs

$8 = k \cdot \frac{3}{2} \cdot \frac{1}{12} \Rightarrow k = \frac{8 \cdot 24}{3} = 64$

$\frac{1}{4}y'' + \gamma y' + 64y = 0$ $\qquad y'' + 4\gamma y' + 256y = 0$

$r^2 + 4\gamma r + 256 = 0$

$(4\gamma)^2 - 4(256) = 16\gamma^2 - 1024 = 0$

$\gamma^2 = 64 \Rightarrow \gamma = 8$

for critical damping

4. $m = 5\text{kg}$ $\qquad k = \frac{5 \cdot 9.8}{.1} = 490$

$\gamma = \frac{2}{.04} = 50$ $\qquad g(t) = 10\sin(t/2)$ $\qquad y(0) = 0,\ y'(0) = .03$

$5y'' + 50y' + 490y = 10\sin(t/2)$

$y'' + 10y' + 98y = 2\sin(t/2)$

$r^2 + 10r + 98 = 0$ $\qquad \frac{-10 \pm \sqrt{100 - 4(98)}}{2} = \frac{-10 \pm \sqrt{292}\,i}{2} = \frac{-10 \pm 2\sqrt{73}i}{2}$

$r = -5 \pm \sqrt{73}\,i$

$y_c(t) = c_1 e^{-5t}\cos\sqrt{73}t + c_2 e^{-5t}\sin\sqrt{73}t$

$Y(t) = A\cos(t/2) + B\sin(t/2)$

$Y'(t) = -\frac{1}{2}A\sin(t/2) + \frac{1}{2}B\cos(t/2)$

$Y''(t) = -\frac{1}{4}A\cos(t/2) - \frac{1}{4}B\sin(t/2)$

$-\frac{1}{4}A\cos(t/2) - \frac{1}{4}B\sin(t/2) - 5A\sin(t/2) + 5B\cos(t/2) + 98A\cos(t/2) + 98B\sin(t/2) = 10\sin(t/2)$

$\cos(t/2)$: $-\frac{1}{4}A + 5B + 98A = 0 \Rightarrow 5B + \frac{391}{4}A = 0 \Rightarrow B = \frac{-391}{20}A$

$\sin(t/2)$: $-\frac{1}{4}B - 5A + 98B = 2 \Rightarrow -5A + \frac{391}{4}\left(\frac{-391}{20}A\right) = 2 \Rightarrow \frac{-153281}{80}A = 2$

$A = \frac{-160}{153281} \approx -.00104$

4 cont'd

$$B = \frac{8}{-100} \cdot \frac{-391}{32} = \frac{3128}{153281} \approx .0204$$

$$y_p(t) = -.00104\cos(t/2) + .0204\sin(t/2)$$

$$y(t) = c_1 e^{-5t}\cos\sqrt{73}t + c_2 e^{-5t}\sin\sqrt{73}t - .00104\cos(t/2) + .0204\sin(t/2)$$

$$0 = c_1(1)(1) + c_2(1)(0) - .00104(1) + .0204(0)$$

$$c_1 = .00104$$

$$y'(t) = -5c_1 e^{-5t}\cos\sqrt{73}t - \sqrt{73}c_1 e^{-5t}\sin(\sqrt{73}t) - 5c_2 e^{-5t}\sin(\sqrt{73}t) + \sqrt{73}c_2 e^{-5t}\cos(\sqrt{73}t) + \frac{.00104}{2}\sin(t/2) + .0102\cos(t/2)$$

$$.03 = -5(.00104)(1)(1) - \sqrt{73}(.00104)(1)(0) - 5c_2(1)(0) + \sqrt{73}c_2(1)(1) + \frac{.00104}{2}(0) + .0102(1)$$

$$.03 - .0102 + 5(.00104) = \sqrt{73}c_2 \Rightarrow \frac{.025}{\sqrt{73}} = \frac{\sqrt{73}c_2}{\sqrt{73}}$$

$$c_2 = .002926\ldots$$

$$y(t) = .00104e^{-5t}\cos\sqrt{73}t + .0029e^{-5t}\sin\sqrt{73}t - .00104\cos(t/2) + .0204\sin(t/2)$$

5. $m = \frac{6}{32} = \frac{3}{16}$ slugs $k = 1$ lbs/in $= 12$ lbs/ft $y(0) = 0$, $y'(0) = 0$

$f(t) = 4\cos(7t)$ $\gamma = 0$

$$\frac{3}{16}y'' + 12y = 4\cos 7t \Rightarrow 3y'' + 192y = 64\cos 7t$$

$$3r^2 + 192 = 0 \Rightarrow r^2 + 64 = 0 \Rightarrow r = \pm 8i$$

$$y_c(t) = c_1\cos 8t + c_2\sin 8t$$

$$Y(t) = A\cos 7t + B\sin 7t$$

$$Y'(t) = -7A\sin 7t + 7B\cos 7t$$

$$Y''(t) = -49A\cos 7t - 49B\sin 7t$$

5 cont'd

$-147A\cos 7t - 147B\sin 7t + 192A\cos 7t + 192B\sin 7t = 64\cos 7t$ ④

$\cos 7t$: $45A = 64 \Rightarrow A = \frac{64}{45}$

$\sin 7t$: $45B = 0 \Rightarrow B = 0$

$y_p(t) = \frac{64}{45}\cos 7t$

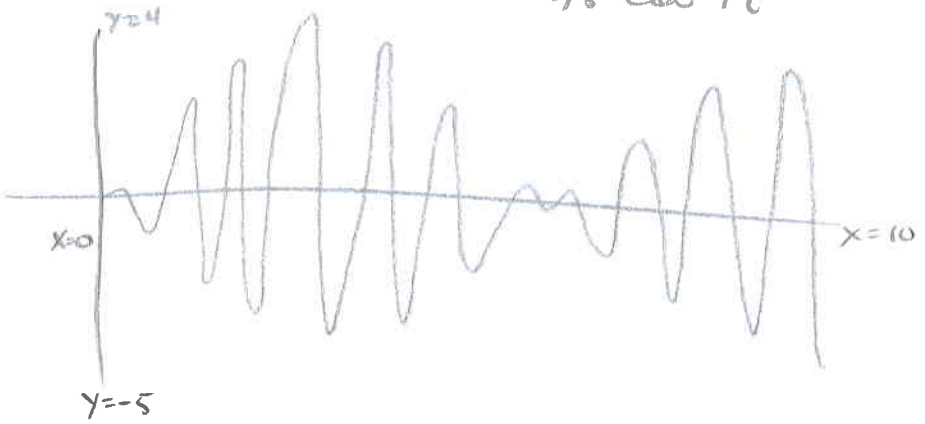
$y(t) = c_1\cos 8t + c_2\sin 8t + \frac{64}{45}\cos 7t$

$0 = c_1(1) + c_2(0) + \frac{64}{45}(1)$ $\quad c_1 = -\frac{64}{45}$

$y'(t) = +\frac{64}{45}\cdot 8\sin 8t + 8c_2\cos 8t - \frac{64}{45}\cdot 7\sin 7t$

$0 = \frac{256}{45}(0) + 8c_2(1) - \frac{448}{45}(0) \Rightarrow c_2 = 0$

$y(t) = -\frac{64}{45}\cos 8t + \frac{64}{45}\cos 7t$



this graph exhibits beats

6. $k = 3$, $m = 2$ $\gamma = 2$ $g(t) = 3\cos 3t$

$2y'' + 2y' + 3y = 3\cos 3t$

$2r^2 + 2r + 3 = 0$ $\quad r = \frac{-2 \pm \sqrt{4 - 4(2)(3)}}{4} = \frac{-2 \pm \sqrt{20}\,i}{4} = \frac{-2 \pm 2\sqrt{5}\,i}{4}$

$r = -\frac{1}{2} \pm \frac{\sqrt{5}}{2}i$

$y_c(t) = c_1 e^{-\frac{1}{2}t}\cos\left(\frac{\sqrt{5}}{2}t\right) + c_2 e^{-\frac{1}{2}t}\sin\left(\frac{\sqrt{5}}{2}t\right)$ $\quad$ not $\neq 3$

$Y(t) = A\cos 3t + B\sin 3t$ $\quad Y'(t) = -3A\sin 3t + 3B\cos 3t$

$Y''(t) = -9A\cos 3t - 9B\sin 3t$

$-18A\cos 3t - 18B\sin 3t - 6A\sin 3t + 6B\cos 3t + 3A\cos 3t + 3B\sin 3t = 3\cos 3t$

$\cos 3t$: $-18A + 6B + 3A = 3 \Rightarrow -15A + 6B = 3 \Rightarrow -5A + 2B = 1$

6 cont'd

$\sin 3t = -18B - 6A + 3B = 0 \Rightarrow -15B - 6A = 0 \Rightarrow \frac{6A}{6} = \frac{-15B}{6}$

$A = -\frac{5}{2}B$

$-5\left(-\frac{5}{2}B\right) + 2B = 1 \Rightarrow \frac{25}{2}B + 2B = \frac{29}{2}B = 1 \Rightarrow B = \frac{2}{29}$

$\Rightarrow A = -\frac{5}{2}\cdot\frac{2}{29} = -\frac{5}{29}$

$y_p(t) = -\frac{5}{29}\cos 3t + \frac{2}{29}\sin 3t$ steady state solution