Math 2568, Quiz #7, Spring 2013 Name KEY

Instructions: Show all work. Give exact answers. All answers must be fully justified to receive full credit.

1. For each of the sets below, determine if the set represents a vector space. If it does, prove it by testing all three conditions for a subspace. If it does not, find at least one case where the vector space conditions are violated.

   a. The set of complex numbers $\mathbb{C}$, in the form $a+bi$, where a,b are real numbers.

① if $a=b=0$ Then $a+bi=0$ ✓

② $(a+bi)+(c+di) = (a+c)+(b+d)i$, $a+c$, $b+d$ real ✓

③ $k(a+bi) = (ka)+(kb)i$ ka real, kb real ✓

is a vector space.

   b. $H = \left\{ \begin{bmatrix} a \\ b \\ c \end{bmatrix}, a+b+c=0 \right\}$ ① $a,b,c=0 \Rightarrow \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ ✓

② $\begin{bmatrix} a \\ b \\ c \end{bmatrix} + \begin{bmatrix} d \\ e \\ f \end{bmatrix} = \begin{bmatrix} a+d \\ b+e \\ c+f \end{bmatrix}$ $(a+b+c)+(d+e+f) = 0+0=0$ ✓

③ $k\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} ka \\ kb \\ kc \end{bmatrix} \Rightarrow ka+kb+kc = k(a+b+c) = k(0)=0$ ✓

or write as span $\begin{bmatrix} a \\ b \\ c \end{bmatrix} = b\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} + c\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$ is a subspace

   c. $W = \left\{ \begin{bmatrix} a & b & 1 \\ a & c & d \end{bmatrix} \right\}$ ① if $a=b=c=d=0$ $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \neq 0$ matrix

not a subspace

   d. The set of polynomials of less than or equal to degree 4 of the form $p(t) = a_1t + a_2t^2 + a_4t^4$ as a subspace of $\mathbb{P}_4$. isomorphic to $\mathbb{R}^5$

$\begin{bmatrix} 0 \\ a_1 \\ a_2 \\ 0 \\ a_4 \end{bmatrix} = a_1\begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + a_2\begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} + a_4\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$ span $\Rightarrow$ subspace

① or if $a_1=a_2=a_4=0$ $p(t)=0$

② $(at+bt^2+ct^4)+(dt+et^2+ft^4) = (a+d)t+(b+e)t^2+(c+f)t^4$
$(a+d), (b+e), (c+f)$ are real

③ $k(at+bt^2+ct^4) = (ka)t+(kb)t^2+(kc)t^4$, ka, kb, kc are real $\Rightarrow$ subspace.