Math 2568, Quiz #14, Spring 2013 Name KEY

Instructions: Show all work. Give exact answers whenever possible.

1. Define an inner product on the vector space of functions by $f \cdot g = \int_{-1}^{1} f(t)g(t)dt$. Determine if the functions $p(t) = 1 - t + t^2$, and $q(t) = -1 - t^2 + t^3$ are orthogonal.

$$\int_{-1}^{1} (1-t+t^2)(-1-t^2+t^3)\,dt =$$

$$\int_{-1}^{1} -1-t^2+t^3+t+t^3-t^4-t^2-t^4+t^5\,dt$$

$$\int_{-1}^{1} -1+t-2t^2+2t^3-2t^4+t^5\,dt =$$

$$\int_{-1}^{1} -1-2t^2-2t^4\,dt + \cancel{\int_{-1}^{1} t+2t^3+t^5\,dt} \to 0$$

$$= 2\int_{0}^{1} -1-2t^2-2t^4\,dt = -t-\tfrac{2}{3}t^3-\tfrac{2}{5}t^5\Big|_0^1 = -1-\tfrac{2}{2}-\tfrac{2}{5} \neq 0$$

no, not orthogonal

2. Find an orthogonal basis for R^3 if one of the vectors is $\vec{u} = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$.

$$V = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} \qquad \vec{u}\cdot\vec{v} = v_1 - 2v_2 + v_3 = 0 \qquad \text{let } v_3 = 0$$

$$\Rightarrow \quad v_1 = 2v_2 \qquad \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} = \vec{v}$$

$$\vec{w} = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} \qquad \vec{u}\cdot\vec{w} = w_1 - 2w_2 + w_3 = 0$$

$$\vec{v}\cdot\vec{w} = 2w_1 + w_2 = 0$$

$$\begin{bmatrix} 1 & -2 & 1 \\ 2 & +1 & 0 \end{bmatrix} \Rightarrow \quad w_1 = -\tfrac{1}{5}w_3, \quad w_2 = \tfrac{2}{5}w_3 \qquad \begin{bmatrix} -1 \\ 2 \\ 5 \end{bmatrix} = \vec{w}$$

Check: $\vec{u}\cdot\vec{v} = 2-2+0=0$

$\vec{u}\cdot\vec{w} \quad -1-4+5=0$ ✓ all orthogonal

$\vec{v}\cdot\vec{w} = -2+2+0=0$

$$A = \begin{bmatrix} 1 & 2 & -1 \\ -2 & 1 & 2 \\ 1 & 0 & 5 \end{bmatrix} \Rightarrow \text{reduces to } I \text{ so is a basis}$$