Math 2568, Quiz #11, Spring 2013 Name KEY

Instructions: Show work to receive partial credit on wrong answers. Decimals are acceptable for question #1, but convert to fractions if your calculator can.

1. A market research firm has been studying the buying pattern for shoppers of three competing products. The stochastic matrix describing their buying habits is given by $P = \begin{bmatrix} .8 & .2 & .05 \\ .05 & .75 & .05 \\ .15 & .05 & .90 \end{bmatrix}$. Find the equilibrium vector for the sales distribution of the products in the long term. How much market share will Product B have?

by calculator $A^{90} = \begin{bmatrix} .3 & .3 & .3 \\ 1/6 & 1/6 & 1/6 \\ 8/15 & 8/15 & 8/15 \end{bmatrix}$ $\vec{q} = \begin{bmatrix} 3/10 \\ 1/6 \\ 8/15 \end{bmatrix}$

by computation

$$\begin{bmatrix} .8-1 & .2 & .05 \\ .05 & .75-1 & .05 \\ .15 & .05 & .9-1 \end{bmatrix} = \begin{bmatrix} -.2 & .2 & .05 \\ .05 & -.25 & .05 \\ .15 & .05 & -.1 \end{bmatrix} \text{rref} \Rightarrow \begin{bmatrix} 1 & 0 & -9/16 \\ 0 & 1 & -5/16 \\ 0 & 0 & 0 \end{bmatrix}$$

$X_1 - 9/16 X_3 = 0$
$X_2 - 5/16 X_3 = 0$
$\Rightarrow$ $X_1 = 9/16 X_3$, $X_2 = 5/16 X_3$, $X_3 = X_3$
$\Rightarrow \vec{q} = \begin{bmatrix} 9 \\ 5 \\ 16 \end{bmatrix} \div 30 \begin{bmatrix} 3/10 \\ 1/6 \\ 8/15 \end{bmatrix} = \vec{q}$

$9+5+16=30$

2. Determine if the vector $\vec{v} = \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}$ is an eigenvector of the matrix $A = \begin{bmatrix} 5 & 4 & 2 \\ 4 & 5 & 2 \\ 2 & 2 & 2 \end{bmatrix}$.

$$\begin{bmatrix} 5 & 4 & 2 \\ 4 & 5 & 2 \\ 2 & 2 & 2 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 5\cdot2 + 4\cdot2 + 2\cdot1 \\ 4\cdot2 + 5\cdot2 + 2\cdot1 \\ 2\cdot2 + 2\cdot2 + 2\cdot1 \end{bmatrix} = \begin{bmatrix} 10+8+2 \\ 8+10+2 \\ 4+4+2 \end{bmatrix} = \begin{bmatrix} 20 \\ 20 \\ 10 \end{bmatrix}$$

$$= 10 \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}$$

Therefore $\begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}$ is an eigenvector of the matrix A; it has eigenvalue $\lambda = 10$