Math 2568, Quiz #10, Spring 2013 Name KEY

Instructions: If the problem cannot be done by inspection, you should show work to justify your answers.

1. For the matrix $A = \begin{bmatrix} 1 & 1 & 0 & -1 \\ -2 & 1 & 0 & 0 \\ 3 & 2 & 0 & -1 \\ -1 & 0 & 1 & 1 \end{bmatrix}$, determine the following:
   a. The Rank of the matrix. 4
   b. The dimensions of Nul A. 0
   c. The dimensions of Row A. 4
   d. The dimensions of Nul A^T. 0
   e. The rank of A^{-1} if it exists. 4

2. For a 9x5 matrix with three pivots find: $n = 5$
   a. Dimensions of Nul A 2
   b. Dimensions of Col A 3
   c. Is the matrix one-to-one? no (not independent - free variables)
   d. Is the matrix onto? no (needs 9 pivots 3 this is impossible since $m > n$)
   e. What are the dimensions of the vector space A maps from? 5 ($\mathbb{R}^5$)
   f. What are the dimensions of the vector space A maps into? 9 (a subspace of $\mathbb{R}^9$)

3. Given the bases B={b_1, b_2, b_3} and C={c_1, c_2, c_3} below, find the change of basis matrices $\underset{C \leftarrow B}{P}$ and $\underset{B \leftarrow C}{P}$. If the B-coordinate vector for $\vec{x}$ is as shown, find the C-coordinate vector for $\vec{x}$.

$$\vec{b_1} = \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}, \vec{b_2} = \begin{bmatrix} 2 \\ 0 \\ 8 \end{bmatrix}, \vec{b_3} = \begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix}, \vec{c_1} = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}, \vec{c_2} = \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}, \vec{c_3} = \begin{bmatrix} 0 \\ -1 \\ -2 \end{bmatrix}, [\vec{x}]_B = \begin{bmatrix} 1 \\ 0 \\ -9 \end{bmatrix}$$

$P_B = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 0 & -1 \\ 3 & 8 & 3 \end{bmatrix}$ $P_C = \begin{bmatrix} 2 & 1 & 0 \\ -1 & 3 & -1 \\ 4 & 5 & -2 \end{bmatrix}$

$P_C[x]_C = P_B[x]_B \Rightarrow$

$[x]_C = P_C^{-1} P_B [x]_B$

$P_C^{-1} P_B = \underset{C \leftarrow B}{P}$

$= \begin{bmatrix} 1/4 & 5/4 & 3/4 \\ 1/2 & -1/2 & -1/2 \\ 1/4 & -11/4 & -5/4 \end{bmatrix}$

$x_B = P_B^{-1} P_C [x]_C$

$P_B^{-1} P_C = \underset{B \leftarrow C}{P} = \begin{bmatrix} 3/2 & 1 & 1/2 \\ -1 & 1 & -1 \\ 5/2 & -2 & 3/2 \end{bmatrix}$

$[x]_C = P_{C \leftarrow B} [x]_B = \begin{bmatrix} -13/2 \\ 5 \\ 23/2 \end{bmatrix}$