Differential Equations, Types Key

I. a. ordinary
b. ordinary
c. ordinary
d. partial
e. partial
f. ordinary
g. ordinary

II. h. first order
i. 2nd order
j. 3rd order
k. second order
l. second order
m. second order
n. second order

III. o. non linear
p. linear
q. linear
r. linear
s. non-linear
t. non-linear
u. non-linear

IV. a. ordinary first-order non-linear D.E.
b. second order linear ordinary D.E.
c. third order linear ODE
d. second order linear partial D.E
e. second order non-linear PDE
f. second order non-linear ODE
g. second order non-linear ODE

V. v. $y = e^{rt}$
w. $y = e^{rt}$
x. $y = e^{rt}$
y. $y = e^{rt}$
z. $\sum c_n (t - t_0)^n$ (series)
aa. $y = x^r$
bb. $y = e^{rt}$
cc. $y = e^{rt}$
dd. $y = x^r$
ee. $y = x^r$
ff. $y = x^r$
gg. $y = e^{rt}$

hh. $y = e^{rt}$
ii. $\sum c_n (x - x_0)^n$ (series)
jj. $\sum c_n (x - x_0)^n$ (series)
kk. $\sum c_n (x - x_0)^n$ (series)
ll. $\sum c_n (x - x_0)^n$ (series)
mm. $\sum c_n (x - x_0)^n$ (series)
nn. $\sum c_n (x - x_0)^n$ (series)
oo. $\sum c_n (x - x_0)^n$ (series)

VI. pp. $M_y = \frac{1}{x}$ $N_x = \frac{1}{x}$ yes

qq. $M_y = 2(x+y)$ $N_x = 2y + 2x$ yes

rr. $M_y = 4$ $N_t = 4$ yes

ss. $M_y = 2y\cos x + 3x^2$ $N_x = 2y\cos x - 3x^2$ NO

tt. $M = e^{2x} + y - 1$ $M_y = 1$ $N = -1$ $N_x = 0$

$(e^{2x} + y - 1)\,dx - dy = 0$

NO

$\frac{M_y - N_x}{N}\mu = \frac{1-0}{-1}\mu = -\mu = \frac{d\mu}{dx}$ $\int -1\,dx = \int \frac{d\mu}{\mu} = -x = \ln\mu$

$\mu = e^{-x}$

$(e^{x} + e^{-x}y - e^{-x})\,dx - e^{-x}\,dy = 0$

$(\mu M)_x = e^{-x}$ $(\mu N)_y = e^{-x}$ now is exact

uu. $M_y = 6x$ $N_x = 18x$ not exact.

$\frac{6x - 18x}{4y + 9x^2}$ not a function of one variable

$\frac{N_x - M_y}{M} = \frac{18x - 6x}{6xy} = \frac{12x}{6xy} = \frac{2}{y}\mu = \frac{d\mu}{dy}$ $\int \frac{2}{y}\,dy = \int \frac{d\mu}{\mu}$

$2\ln y = \ln\mu = \ln y^2$ $\mu = y^2$

$6xy^3\,dx + (4y^3 + 9x^2y^2)\,dy = 0$

$(\mu M)_y = 18xy^2$ $(\mu N)_x = 18xy^2$ now it is exact

vv. $M_y = -6$ $N_x = 0$ not exact

$\frac{-6-0}{2} = -3 \Rightarrow -3\mu = \frac{d\mu}{dx} \Rightarrow -3\,dx = \frac{d\mu}{\mu} \Rightarrow -3x = \ln\mu \Rightarrow \mu = e^{-3x}$

$(10e^{-3x} - 6e^{-3x}y + e^{-6x})\,dx + 2e^{-3x}\,dy = 0$

$(\mu M)_y = -6e^{-3x}$ $(\mu N)_x = -6e^{-3x}$ now it is exact ww. $(x^2+y^2-5)\,dx-(y+xy)\,dy=0$

$M_y=2y \qquad N_x=-y \qquad$ not exact

$$\frac{2y-(-y)}{-y+xy}=\frac{3\cancel{y}}{-(\cancel{y})(1+x)}=-\frac{3}{1+x} \Rightarrow \frac{-3}{1+x}\mu=\frac{d\mu}{dx} \Rightarrow \int\frac{-3}{1+x}\,dx=\int\frac{d\mu}{\mu}$$

$$-3\ln|1+x|=\ln\mu \Rightarrow \mu=(1+x)^{-3}$$

$$\frac{(x^2+y^2-5)}{(1+x)^3}\,dx-\frac{y\cancel{(1+x)}}{(1+x)^{\cancel{3}\,2}}\,dy \Rightarrow \frac{x^2+y^2-5}{(1+x)^3}\,dx-\frac{y}{(1+x)^2}\,dy=0$$

$$(\mu M)_y=\frac{2y}{(1+x)^3} \qquad (\mu N)_x=-\frac{y\cdot -2}{(1+x)^3}=\frac{2y}{(1+x)^3}$$ now it is exact

VII.

xx. $p(t)=\frac{2}{t} \quad g(t)=\frac{\cos t}{t^2}$

yy. $p(t)=\frac{t+1}{t} \quad g(t)=1 \qquad y'+\frac{t+1}{t}y=1$

zz. $p(x)=4 \quad g(x)=\frac{4}{3} \qquad y'+4y=\frac{4}{3}$

a. $p(x)=-\frac{1}{x} \quad g(x)=x\sin x \qquad y'-\frac{1}{x}y=x\sin x$

b. $p(t)=-2 \quad g(t)=t \qquad y'-2y=t$

c. $p(t)=\frac{4t}{1+t^2} \quad g(t)=(1+t^2)^{-3} \qquad y'+\frac{4t}{1+t^2}y=(1+t^2)^{-3}$

VIII. d. $\left(y'+2xy=xy^2\right)(-1y^{-2}) \qquad z_1=y^{1-n}=y^{-1}$

$$-y^{-2}y'-2xy^{-1}=-x \qquad \frac{dz}{dx}=-1y^{-2}y'$$

$$z'-2xz=-x$$

$p(x)=-2x \quad g(x)=-x$ linear now

e. $xy'+y=xy^3 \qquad 1-n=-2 \qquad * -2y^{-3}$

$$\frac{-2\cancel{x}y^{-3}y'}{\cancel{x}}+\frac{-2y^{-2}}{x}=\frac{-2\cancel{x}}{\cancel{x}}$$

$$z=y^{-2}$$

$$-2y^{-3}y'-\frac{2}{x}y^{-2}=-2 \qquad z'=-2y^{-3}y'$$

VIII e cont'd

$z' - \frac{2}{x} z = -2 \qquad p(x) = -\frac{2}{x} \quad g(x) = -2$ now linear

f. $\frac{yy'}{y} + \frac{\frac{1}{x}y^2}{y} = \frac{x\sqrt{y}}{y} \Rightarrow y' + \frac{1}{x}y = x y^{-1/2} \qquad n = -1/2 \qquad * \frac{3}{2} y^{1/2}$

$\frac{3}{2} y^{1/2} y' + \frac{3}{2}\frac{1}{x} y^{3/2} = \frac{3}{2} x \qquad z = y^{3/2} \quad z' = \frac{3}{2} y^{1/2} y'$

$z' + \frac{3}{2x} z = \frac{3x}{2}$ now linear $\quad p(x) = \frac{3}{2x}, \; g(x) = \frac{3x}{2}$

g. $\frac{xy'}{x} + \frac{y}{x} = \frac{\frac{1}{y^2}}{x} \Rightarrow y' + \frac{1}{x} y = \frac{1}{x} y^{-2} \qquad 1-n=3 \qquad * 3y^2$

$3y^2 y' + \frac{1}{x} 3y^3 = \frac{1}{x} \qquad z = y^3 \quad z' = 3y^2 y'$

$z' + \frac{3}{x} z = \frac{1}{x} \qquad p(x) = \frac{3}{x} \quad g(x) = \frac{1}{x}$ now linear

h. $y' = y(xy^3 - 1) = xy^4 - y \Rightarrow y' + y = xy^4 \qquad n = 4$

$* -3y^{-4}$

$-3y^{-4} y' + -3y^{-3} = x \qquad z = y^{-3} \quad z' = -3y^{-4} y'$

$z' + -3z = x \qquad p(x) = -3 \quad g(x) = x$ now linear

XI. i. can be made linear

ii. bernoulli

iii. homogeneous

iv. exact

v. separable

vi. linear

vii. bernoulli $\quad y' - 2y = e^x y^{-1}$

viii. separable

ix. homogeneous

x. separable

xi. $M_y = \ln y - 1 + \ln x \qquad N_x = \ln x + 1 - \ln y$ numerical

xii. $\frac{(6x+1)y^2 y'}{y^2} + \frac{3x^2}{y^2} = \frac{2y^3}{y^2}$

$(6x+1)y' + \frac{3x^2}{y^2} = 2y \Rightarrow (6x+1)y' - 2y = -3x^2y^{-2}$

bernoulli.

xiii. $(3y^2+2x)dx + (4y^2+6xy)\,dy = 0$

$M_y = 6y \qquad N_x = 6y$ exact

xiv $1\,dx - (2x+y+1)\,dy = 0$

$M_y = 0 \quad N_x = -2 \qquad \frac{N_x - M_y}{M} =$ not exact

$\frac{-2-0}{1} = -2\mu = \frac{d\mu}{dy} \Rightarrow \int -2\,dy = \int \frac{d\mu}{\mu} \Rightarrow -2y = \ln\mu \qquad \mu = e^{-2y}$

$e^{-2y}dx - e^{2y}(2x+y-1)\,dy = 0$

$(\mu M)_y = -2e^{-2y} \qquad (\mu N)_x = -2e^{-2y}$ now exact

xv. $(x^2+4)\,dy = 2x(1-4y)\,dx$

$\frac{dy}{1-4y} = \frac{2x}{x^2+4}\,dx$ Separable

xvi. $\frac{y}{x^2}\frac{dy}{dx} + e^{2x^3+y^2} = 0 \Rightarrow \frac{y}{x^2}dy = -e^{2x^3}e^{y^2}dx \Rightarrow$

$ye^{-y^2}dy = -x^2e^{2x^3}\,dx$ separable

xvii. $\frac{dy}{dx} = \frac{x^2+y^2+xy}{xy}$ homogeneous.