Math 2255, Quiz #7, Spring 2013 Name KEY

1. Use undetermined coefficients to find the general solution to $y'' + 4y = 5\sin(t)$.

$r^2+4=0 \quad r=\pm 2 \quad y_c(t) = C_1\cos 2t + C_2 \sin 2t$

$Y(t) = A\cos t + B\sin t$

$Y'(t) = -A\sin t + B\cos t$

$Y''(t) = -A\cos t - B\sin t$

$-A\cos t - B\sin t + 4A\cos t + 4B\sin t = 5\sin t$

$3A\cos t = 0\cos t \Rightarrow A = 0$

$3B\sin t = 5\sin t \Rightarrow B = 5/3$

$Y_p(t) = 5/3 \sin t$

$\boxed{y(t) = C_1\cos 2t + C_2 \sin 2t + 5/3 \sin t}$

2. Use variation of parameters to find the general solution to $y'' + 4y = 5\csc(2t)$.

$y_1 = \cos 2t, \quad y_2 = \sin 2t$

$W = \begin{vmatrix} \cos 2t & \sin 2t \\ -2\sin 2t & 2\cos 2t \end{vmatrix} = 2\cos^2 2t + 2\sin^2 2t = 2$

$y(t) = -\cos 2t \int \frac{\sin 2t \cdot 5\csc 2t}{2}\,dt + \sin 2t \int \frac{\cos 2t \csc 2t}{2}\,dt$

$= -\cos 2t \int \frac{5}{2}\,dt + \frac{1}{2}\sin 2t \int \cot 2t\,dt =$

$\boxed{-\frac{5}{2}\cos 2t\,(t) + C_1\cos 2t + \frac{1}{4}\sin 2t \ln|\sin 2t| + C_2 \sin 2t}$