Math 2255, Quiz #4, Spring 2013 Name KEY

Instructions: Show all work. Be sure to solve each equation to the end. Solve for any constants if initial conditions are provided. Use exact answers unless specifically asked to round.

1. Solve the Bernoulli equation $y' + \frac{3}{x}y = \frac{4}{x}e^{-2x}y^{\frac{4}{3}}$, y(1)=2.

$n = 4/3 \quad (1-n)y^{-n} = -\frac{1}{3}y^{-4/3}$

$-\frac{1}{3}y^{-4/3}y' + \frac{3}{x}\left(-\frac{1}{3}y^{-1/3}\right) = \frac{4}{x}\left(-\frac{1}{3}\right)e^{-2x}$

$-\frac{1}{3}y^{-4/3}y' - \frac{1}{x}y^{-1/3} = \frac{-4}{3x}e^{-2x}$

$z = y^{-1/3} \quad z' = -\frac{1}{3}y^{-4/3}y'$

$z' - \frac{1}{x}z = \frac{-4}{3x}e^{-2x}$

$\mu = e^{\int -\frac{1}{x}dx} = e^{-\ln x} = \frac{1}{x}$

$\frac{1}{x}z' - \frac{1}{x^2}z = \frac{-4}{x^2}e^{-2x}$

$\int \left(\frac{1}{x}z\right)' = \int \frac{-4}{x^2}e^{-2x}\,dx$

$z = x\left[\int \frac{-4}{x^2}e^{-2x}\,dx + C\right]$

this has to be integrated by series, so you can stop here.

$y^{-1/3} = x\left[\int \frac{-4}{x^2}e^{-2x}\,dx + C\right]$

$\boxed{y = x^{-3}\left[\int \frac{-4}{x^2}e^{-2x}\,dx + C\right]^{-3}}$

2. Solve the exact equation
$\left(3x^2y - 4xy^2 - e^x\sin y - \frac{1}{x+1}\right)dx + (x^3 - 4x^2y - e^x\cos y + 1)dy = 0$, y(0)=0.

$M_y = 3x^2 - 8xy - e^x\cos y$

$N_x = 3x^2 - 8xy - e^x\cos y$ ✓

$\int 3x^2y - 4xy^2 - e^x\sin y - \frac{1}{x+1}\,dx = x^3y - 2x^2y^2 - e^x\sin y - \ln|x+1| + f(y)$

$\int x^3 - 4x^2y - e^x\cos y + 1\,dy = x^3y - 2x^2y^2 - e^x\sin y + y + g(x)$

$\psi(x,y)$: $x^3y - 2x^2y^2 - e^x\sin y - \ln|x+1| + y = c$

$0 \quad 0 \quad 0 \quad 0 \quad 0 = c \qquad c = 0$

$\boxed{x^3y - 2x^2y^2 - e^x\sin y - \ln|x+1| + y = 0}$