Math 2255, Quiz #15, Spring 2013 Name KEY

Instructions: Show all work. Give at least 4 terms for each coefficient a_0, a_1 for the solution to the series.

1. Solve the differential equation using series solution methods $2y'' - (x+1)^2y' + xy = 0$ centered at $x_0 = -1$.

$(x+1)y - y$

$$2\sum_{n=2}^{\infty} a_n n(n-1)(x+1)^{n-2} - (x+1)^2\sum_{n=1}^{\infty} a_n n(x+1)^{n-1} + (x+1)\sum_{n=0}^{\infty} a_n (x+1)^n - \sum_{n=0}^{\infty} a_n (x-1)^n = 0$$

$$\sum_{n=2}^{\infty} 2a_n n(n-1)(x+1)^{n-2} - \sum_{n=1}^{\infty} a_n n(x+1)^{n+1} + \sum_{n=0}^{\infty} a_n (x+1)^{n+1} - \sum_{n=0}^{\infty} a_n (x+1)^n = 0$$

$$\sum_{n=-1}^{\infty} 2a_{n+3}(n+3)(n+2)(x+1)^{n+1} - \sum_{n=1}^{\infty} a_n n(x+1)^{n+1} + \sum_{n=0}^{\infty} a_n (x+1)^{n+1} - \sum_{n=-1}^{\infty} a_{n+1}(x+1)^{n+1} = 0$$

$$2a_2(2)(1)(x+1)^0 + 2a_3(3)(2)(x+1)^1 + \sum_{n=1}^{\infty} 2a_{n+3}(n+3)(n+2)(x+1)^{n+1} - \sum_{n=1}^{\infty} a_n n(x+1)^{n+1}$$
$$+ a_0(x+1)^1 + \sum_{n=0}^{\infty} a_n (x+1)^{n+1} - a_0(x+1)^0 - a_1(x+1)^1 - \sum_{n=1}^{\infty} a_{n+1}(x+1)^{n+1} = 0$$

$$\sum_{n=1}^{\infty}\Big[2a_{n+3}(n+3)(n+2) \underbrace{- a_n n + a_n}_{a_n(1-n)} - a_{n+1}\Big](x+1)^{n+1} = 0$$

$$4a_2 - a_0 = 0 \Rightarrow a_2 = \frac{1}{4}a_0$$

$$12a_3 + a_0 - a_1 = 0 \Rightarrow a_3 = \frac{1}{12}a_1 - \frac{1}{12}a_0$$

$$a_{n+3} = \frac{a_{n+1}}{2(n+3)(n+2)} + \frac{a_n(n-1)}{2(n+3)(n+2)}$$

$n=1$

$$a_4 = \frac{a_2}{2(4)(3)} + \cancel{\frac{a_1(0)}{2(4)(3)}} = \frac{1}{24}\left(\frac{1}{4}a_0\right) = \frac{1}{96}a_0$$

$n=2$

$$a_5 = \frac{a_3}{2(5)(4)} + \frac{a_2(2)}{2(5)(4)} = \frac{1}{40}\left(\frac{1}{12}a_1 - \frac{1}{12}a_0\right) + \frac{1}{20}\left(\frac{1}{4}a_0\right) = \frac{1}{480}a_1 + \frac{1}{96}a_0$$

$n=3$

$$a_6 = \frac{a_4}{2(6)(5)} + \frac{a_3(3)}{2(6)(5)} = \frac{1}{60}\left(\frac{1}{96}a_0\right) + \frac{1}{20}\left(\frac{1}{12}a_1 - \frac{1}{12}a_0\right) = \frac{1}{120}a_1 - \frac{23}{5760}a_0$$

$$y = a_0\left(1 + \frac{1}{4}x^2 - \frac{1}{12}x^3 + \frac{1}{96}x^4 + \frac{1}{96}x^5 + \dots\right) + a_1\left(x + \frac{1}{12}x^3 + \frac{1}{480}x^5 + \frac{1}{120}x^6 + \dots\right)$$