Math 2255, Quiz #12, Spring 2013 Name KEY

**Instructions:** Show all work. You may use the attached tables for the transform formulas.

1. Solve the differential equations using Laplace Transforms.
   a. $y'' + 6y' - 7y = te^t, y(0) = 0, y'(0) = 2$

$$s^2Y(s) - 0 - 2 + 6sY(s) + 0 - 7Y(s) = \frac{1}{(s-1)^2}$$

$$Y(s)(s^2+6s-7) = \frac{1+2(s^2-2s+1)}{(s-1)^2} = \frac{1+2s^2-4s+2}{(s-1)^2} = \frac{2s^2-4s+3}{(s-1)^2}$$

$$(s+7)(s-1)$$

$$Y(s) = \frac{2s^2-4s+3}{(s+7)(s-1)^3} = \frac{A}{s+7} + \frac{B}{(s-1)} + \frac{C}{(s-1)^2} + \frac{D}{(s-1)^3}$$

$$y(t) = Ae^{-7t} + Be^{t} + Cte^{t} + \frac{D}{2}t^2e^{t} = \frac{-129}{512}e^{-7t} + \frac{1863}{8192}e^{t} + \frac{73}{8192}te^{t} + \frac{1}{16}t^2e^{t}$$

$$A(s-1)^3 + B(s+7)(s-1)^2 + C(s+7)(s-1) + D(s+7) = 2s^2-4s+3$$

$s=1$ $\quad 8D = 2-4+3 = 1 \quad D = 1/8$ $\qquad s=-7 \quad -512A = 129 \Rightarrow A = \frac{-129}{512}$

$s=0$ $\quad -A + 8B - 8C + 8D = 3 \Rightarrow \frac{129}{512} + 8B - 8C + 1 = 3 \Rightarrow 8B - 8C = \frac{895}{512} \Rightarrow B - C = \frac{895}{4096}$

$s=2$ $\quad A + 9B + 9C + 9D = 3 \Rightarrow \frac{-129}{512} + 9B + 9C + \frac{9}{8} = 3 \Rightarrow 9B + 9C = \frac{1089}{512} \Rightarrow B + C = \frac{121}{512}$

$$2B = \frac{1863}{4096}$$

$$B = \frac{1863}{8192} \qquad C = \frac{121}{512} - \frac{1863}{8192} = \frac{73}{8192}$$

   b. $y'' - 5y' + 6y = f(t), y(0) = 1, y'(0) = 0$, for $f(t) = \begin{cases} 0, & 0 \le t < 3 \\ 4, & t \ge 3 \end{cases} = 4u_3(t)$

$$s^2Y(s) - s - 0 - 5sY(s) + 5 + 6Y(s) = \frac{4e^{-3s}}{s}$$

$$Y(s)(s^2-5s+6) = \frac{4e^{-3s}}{s} + s \Rightarrow Y(s) = \frac{4e^{-3s}}{s(s-3)(s-2)} + \frac{s}{(s-3)(s-2)}$$

$$(s-3)(s-2)$$

$$= 4e^{-3s}\left[\frac{A}{s} + \frac{B}{s-3} + \frac{C}{s-2}\right] + \left[\frac{D}{s-3} + \frac{E}{s-2}\right]$$

$$y(t) = 4\left[A + Be^{3(t-3)} + Ce^{2(t-3)}\right]u(t-3) + De^{3t} + Ee^{2t}$$

$$A(s-3)(s-2) + Bs(s-2) + C(s)(s-3) = 1 \qquad s=0 \Rightarrow 6A = 1 \Rightarrow A = \frac{1}{6}$$

$s=2 \Rightarrow \quad -2C = 1 \Rightarrow C = -1/2 \qquad s=3 \Rightarrow \quad 3B = 1 \Rightarrow B = 1/3$

$$D(s-2) + E(s-3) = s \qquad s=2 \Rightarrow -E = 2 \quad E = -2 \qquad s=3 \Rightarrow D = 3$$

$$y(t) = \left[\frac{2}{3} + \frac{4}{3}e^{3t-9} - 2e^{2t-6}\right]u(t-3) + 3e^{3t} - 2e^{2t}$$