Math 1116 Frequent Flyer Project (Work) Key

1. I will choose the top 4 for my calculations. Your answers will vary depending on what you chose.

A = NY B = LA C = Chicago D = Houston

a) Brute Force 3 circuits

ABCDA = 2790 + 2016 + 1084 + 1628 = 7518

ABDCA = 2790 + 1546 + 1084 + 790 = 6210

* ACBDA = 790 + 2016 + 1546 + 1628 = 5980 miles optimal

b) Nearest Neighbor (starting at NY)

NY – Chicago – Houston – LA – NY

790 + 1084 + 1546 + 2790 = 6210 miles

percent error: $\frac{6210 - 5980}{5980} = 0.03846 \approx 3.8\%$

c) Repeated Nearest Neighbor

LA – Houston – Chicago – NY – LA

1546 + 1084 + 790 + 2790 = 6210

Chicago – NY – Houston – LA – Chicago

790 + 1628 + 1546 – 2016 = 5980 miles * best

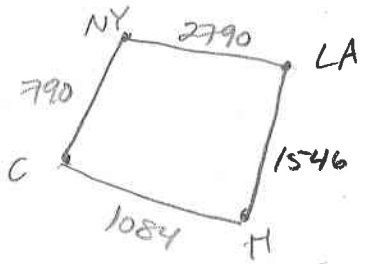
Houston – Chicago – NY – LA – Houston

1084 + 790 + 2790 – 1546 = 6210

percent error: $\frac{5980 - 5980}{5980} = 0\%$

in the B city problem I will use a 1% error to illustrate the calculation since in real life the true error is not likely to ever be 0.

1d) Cheapest Link



= 6210 miles

percent error = $\frac{6210 - 5980}{5980} = 3.8\%$

2. a) Nearest Neighbor

Starting in NY

NY - Philadelphia - Indianapolis - Chicago - Dallas -
97 + 644 + 183 + 968 +

- Austin - San Antonio - Houston - Jacksonville - Phoenix
196 + 80 + 197 + 870 + 2043 +

- San Diego - LA - San Jose - NY
355 + 121 + 340 + 2939 = 9,033 miles

Brute force estimate from this is

$\times \cancel{(1.038)} = \frac{9033}{1.038} \approx$ ~~8702~~ miles

b) Repeated Nearest Neighbor

i) NY above

ii) LA

LA - SD - Phoenix - SJ - Dallas - Austin - SA - Houston - Jacksonville
121 + 355 + 710 + 1688 + 196 + 80 + 197 + 870 +

Philly - NY - Indianapolis - Chicago - LA
850 + 97 + 709 + 183 + 2016 = 8072

2 b) iii) Chicago

Chicago - Indianapolis - Philadelphia - NY - Jacksonville - Houston -
183 + 644 + 97 + 936 + 870 +

- Austin - SA - Dallas - Phoenix - San Diego - LA - SJ - Chicago
+162 + 80 + 274 + 1064 + 355 + 121 + 340 + 2164 =

7280 miles *

iv) Houston

Houston - Austin - SA - Dallas - Indianapolis - Chicago - Philadelphia -
162 + 80 + 274 + 899 + 183 + 759 +

NY - Jacksonville - Phoenix - SD - LA - SJ - Houston
97 + 936 + 2043 + 355 + 121 + 340 + 1884 =

8133

v) Philadelphia

Philly - NY - Indy - Chicago - Dallas - Austin - SA - Houston - Jacksonville
97 + 709 + 183 + 968 + 196 + 80 + 197 + 870 +

- Phoenix - SD - LA - SJ - Philly
2043 + 355 + 121 - 340 + 2908 = 9067

vi) Phoenix

Phoenix - SD - LA - SJ - Dallas - Austin - SA - Houston - Jacksonville
355 + 121 + 340 + 1688 + 196 + 80 + 197 + 870 +

- Philly - NY - Indy - Chicago - Phoenix
850 + 97 + 709 + 183 + 1753 = 7439

vii) San Antonio

SA - Austin - Houston - Dallas - Indy - Chicago - Philly - NY -
80 + 162 + 239 + 899 + 183 + 759 + 97 +

Jacksonville - Phoenix - SD - LA - SJ - SA
936 + 2043 + 355 + 121 + 340 + 1692 = 7706

viii) San Diego

SD – LA – SJ – Phoenix + SA – Austin – Houston – Dallas – Indy – Chicago
121 + 340 + 710 + 982 + 80 + 162 + 239 + 899 + 183 +

– Philly – NY – Jacksonville – SD
759 + 97 + 936 + 2336 = 7844

ix) Dallas

Dallas – Austin – SA – Houston – Jacksonville – Philly – NY – Indy –
196 + 80 + 197 + 870 + 936 + 97 + 709 +

Chicago – Phoenix – SD – LA – SJ – Dallas
183 + 1753 + 355 + 121 + 340 + 1688 = 7525

x) San Jose

SJ – LA – SD – Phoenix – SA – Austin – Houston – Dallas – Indy – Chicago –
340 + 121 + 355 + 982 + 80 + 162 + 239 + 899 + 183 +

Philly – NY – Jacksonville – SJ
759 + 97 + 936 + 2753 = 7906

xi) Jacksonville

Jacksonville – Philly – NY – Indy + Chicago – Dallas – Austin – SA – Houston
850 + 97 + 709 + 183 + 968 + 196 + 80 + 197 +

– Phoenix – SD – LA – SJ – Jacksonville
1174 + 355 + 121 + 340 + 2753 = 8023

xii) Indianapolis

Indy – Chicago – Philly – NY – Jacksonville – Houston – Austin – SA –
183 + 759 + 97 + 936 + 870 + 162 + 80 +

Dallas – Phoenix – SD + LA – SJ – Indy
274 + 1064 + 355 + 121 + 340 + 2050 = 7291

xiii) Austin

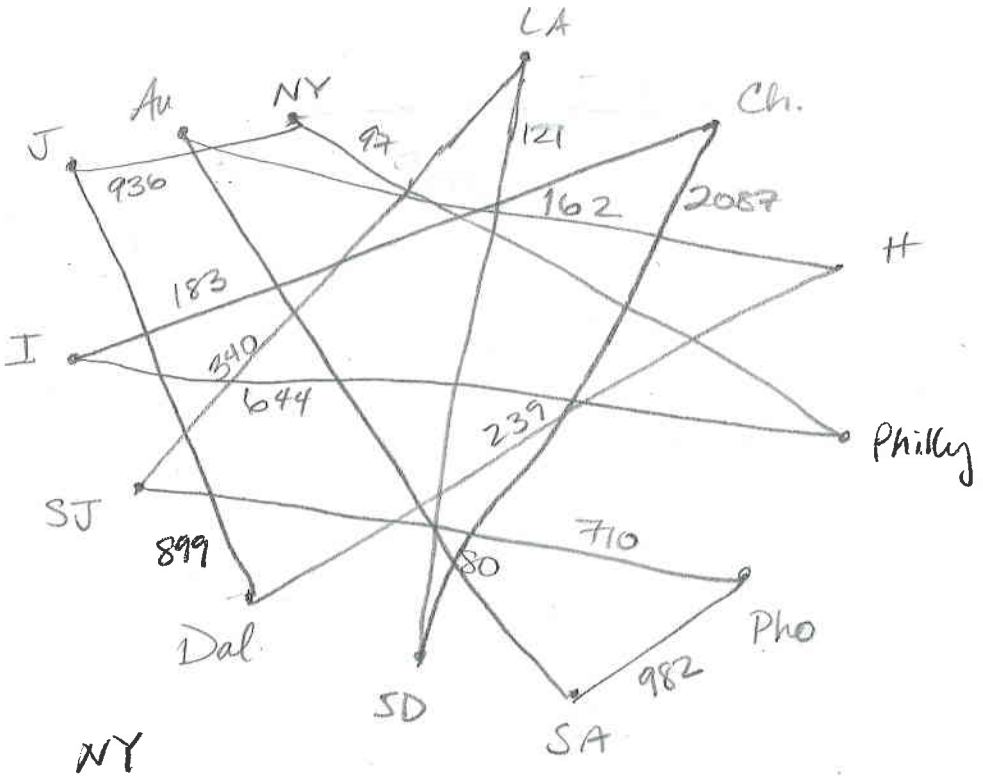
Austin – SA – Houston – Dallas – Indy – Chicago – Philly – NY – Jacksonville –
80 + 197 + 239 + 899 + 183 + 759 + 97 + 936 +

– Phoenix – SD – LA – SJ – Austin
2043 + 355 + 121 + 355 + 1716 = 7980

7280 is best approximation

based on my 4-city case, this is my approximation for The Brute force, but using the 1% figure: $\frac{7980}{1.01} \approx 7208$ miles

c) cheapest link



NY

97 + 644 + 183 + 2087 + 121 + 340 + 710 + 982 + 80 + 162 + 239
+ 899 + 936 = 7480 miles

brute force estimate $\frac{7480}{1.038} \approx 7206$ miles

3. d) for short Bruteforce, we computed all possible circuits before, so just select the largest here:

NY - LA - Chicago - Houston - NY = 7518

a) "Farthest" Neighbor

NY - LA - Chicago - Houston - NY
2790 + 2016 + 1084 + 1628 = 7518

this is ophimal % error = 0

b) Repeated Farthest Neighbor

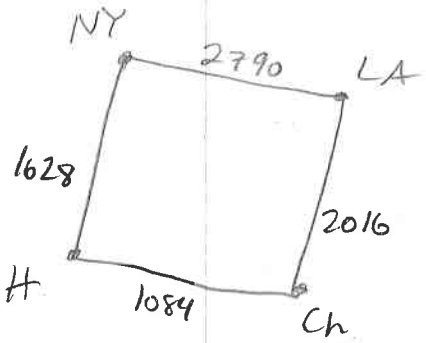
LA - NY - Houston - Chicago - LA
2790 - 1628 + 1084 + 2016 = 7518

Chicago - LA - NY - Houston - Chicago
2016 + 2790 + 1628 + 1084 = 7518

Houston - NY - LA - Chicago - Houston
1628 + 2790 + 2016 + 1084 = 7518

all ophimal percent error = 0

c) Most Expensive Leik



= 7518 ophimal

percent error = 0

3) e) Starting in NY Farthest Neighbor

NY - SJ - Philly - LA - Jacksonville - SD - Chicago - Phoenix -
2939 + 2908 + 2709 + 2415 + 2336 + 2077 + 1753 + 1699

Indy - SA - Dallas - Houston - Austin - NY
+ 1172 + 274 + 239 + 162 + 1743 = 22,426 miles

Estimate for Brute force is the same since short version was exact

f) Repeated Farthest Neighbor

i) NY above

ii) Philadelphia

Philly - SJ - NY - LA - Jacksonville - SD - Indy - Phoenix -
2908 + 2939 + 2790 + 2415 + 2336 + 2055 + 1699 +
Chicago - SA - Dallas - Houston - Austin - Philly
1753 + 1242 + 274 + 239 + 162 + 1662 = 22,474

iii) Chicago

Chicago - SJ - NY - LA - Philly - SD - Jacksonville - Phoenix -
2106 + 2939 + 2790 + 2709 + 2694 + 2336 + 2043 +

Indy - SA - Dallas - Houston - Austin - Chicago
1699 + 1172 + 274 + 239 + 162 + 1164 = 22,327

iv) Houston - SJ - NY - LA - Philly - SD - Jacksonville - Phoenix - Indy -
1884 + 2939 + 2790 + 2709 + 2694 + 2336 + 2043 + 1699 +

SA - Dallas - Austin - Houston
1172 + 274 + 196 - 162 = 20898

v) LA

LA - NY - SJ - Philly - SD - Jacksonville - Phoenix - Chicago - SA -
2790 + 2939 - 2908 + 2694 + 2336 + 2043 + 1753 + 1242

Indy - Austin - Dallas - Houston - LA
1172 + 1095 + 196 + 239 - 1546 = 22953

5. b. vi) Phoenix

Phoenix – NY – SJ – Philly – LA – Jacksonville – SD – Chicago –
2407 – 2908 + 2709 + 2415 + 2336 + 2077 +

SA – Indy – Austin + Dallas – Houston – Phoenix
1242 + 1172 + 1095 + 196 + 239 + 1174 = 19,970

vii) San Antonio

SA – NY – SJ – Philly – LA – Jacksonville – SD – Chicago – Phoenix
1824 + 2939 + 2908 + 2709 + 2415 + 2336 + 2077 + 1753

– Indy – Austin – Dallas – Houston – SA
1699 + 1095 + 196 + 239 + 274 = 22464

viii) San Diego

SD – NY – SJ – Philly – LA – Jacksonville – Phoenix – Chicago – SA –
2760 + 2939 – 2908 + 2709 + 2415 + 2043 + 1753 + 1242 +

Indy – Austin – Dallas – Houston – SD
1172 + 1095 + 196 + 239 + 1468 = 23,939

ix) Dallas

Dallas – SJ – NY – LA – Philly – SD – Jacksonville – Phoenix –
1688 + 2939 + 2790 + 2709 + 2694 + 2336 + 2043

Indy – SA – Chicago – Austin – Houston – Dallas
1699 + 1172 + 1242 + 1162 + 162 + 239 = 22875

x) San Jose

SJ – NY – LA – Philly – SD – Jacksonville – Phoenix – Chicago – SA –
2939 + 2790 + 2709 + 2694 + 2336 + 2043 + 1753 + 1699 +

Indy – Austin – Dallas – Houston – SJ
1172 + 1095 + 196 + 239 + 1884 = 23549 * max

xi) Jacksonville

Jacksonville – San Jose – NY – LA – Philly – SD – Chicago – Phoenix
2753 + 2939 + 2790 + 2709 + 2694 + 2077 + 1753 +

x.) cont'd

- Indy - SA - Dallas - Houston - Austin - Jacksonville
+1699 + 1172 + 274 + 239 + 162 + 1031 = 22,292

xii) Indianapolis

Indy - SJ - NY - LA - Philly - SD - Jacksonville - Phoenix - Chicago -
2280 + 2939 + 2790 + 2709 + 2694 + 2336 + 2043 + 1753 +

SA - Dallas - Houston - Austin - Indy
1099 + 274 + 239 + 162 + 1095 = 23,013

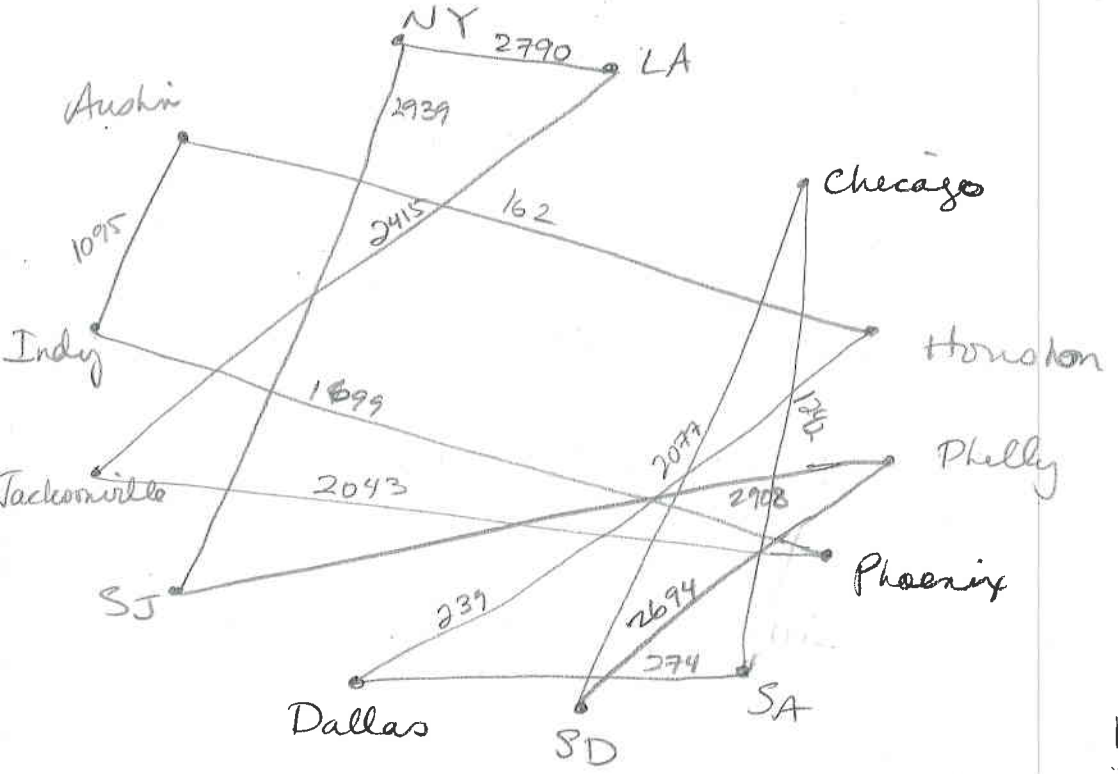
xiii) Austin.

Austin - NY - SJ - Philly - LA - Jacksonville - SD - Chicago - Phoenix
1743 + 2939 + 2908 + 2709 + 2415 + 2336 + 2077 + 1753 + 1699 +

Indy - SA - Dallas Houston - Austin -
1172 + 274 + 239 + 162 = 22426

maximum route this way is 23,549 miles
approx max for brute force is the same since est. error is 0%.

g) Most Expensive Link



= 22,577

since our estimated error was 0%, this is the brute force estimate, but we know from above that it can be higher.

2. questions

There are 239.5 million unique Hamilton circuits for brute force our best estimate using all approximation methods was 7480 leads to approximation of 7206 for brute force: that's a difference of 274 miles or #137. probably not worth the effort here. The cost per mile would need to be significantly higher and you'd need some cheap time or a supercomputer. If you could calculate one circuit per minute, you'd need 456 years to find them all. Even one per second would take nearly 8 years.

3. You appear to be losing ~ 14-15,000 miles by using cheapest instead of longest methods. The longest methods triple the number of miles traveled, and therefore probably the time as well. If flight costs have any relation to miles travelled, its unlikely your boss would go for this and he would certainly complain about it.