Math 1116, Exam #3/Final Exam, Spring 2013 Name KEY

Instructions: Show all work. You should use exact answers unless you are specifically asked to round (this may be often on this exam, but not universally). Incorrect answers that come with no work will not receive partial credit. Be sure to read the instructions clearly!

1. Use Hamilton's Method to apportion 250 among the following five states. Be sure to clearly label the standard divisor, and the total of each column of the lower quota, the upper quota and the final apportionment. You may round decimal answers to 2 digits. (21 points)

State	Population (in millions)	Standard Quota	Lower Quota	Upper Quota	Surplus Seats	Final Apportionment
Romana	40.5	85.084	85	86		85
Selena	29.7	62.395	62	63		62
Tarsia	23.65	49.685	49	50	1	50
Urania	14.6	30.672	30	31	1	31
Vela	10.55	22.164	22	23		22
Totals:	119		248	253		250

Standard Divisor: .476

2. Use Huntington-Hill's Method to apportion 35 employees among the 4 shifts with the given number of patients. As above, be sure to list totals in the same three columns. If the standard quota does not work try a modified divisor of 75 or 80 as needed. You should carry at least 4 digits. (21 points)

Shifts	Patients	Standard Quota	Modified Quota	Lower (Mod) Quota	Upper (Mod) Quota	$\sqrt{LQ * UQ}$	Final Apportionment
Morning	871	11.2907		11	12	11.4891	11
Afternoon	1029	13.3389		13	14	13.4907	13
Evening	610	7.9074		7	8	7.4833	8
Late Night	190	2.4630		2	3	2.4495	3
Totals	2700			33	37		35

Standard Divisor: 77.143

Did you use a modified divisor? If so, which one?

no modified divisor needed

3. The following table calculating an apportionment using Jefferson's method has been completed for you, using 250 seats. It contains a paradox or some other fairness violation we discussed this term. Find it and explain what is wrong. (10 points)

State	Population (in thousands)	Standard Quota (SD=50,000)	Lower Quota	Modified Quota (MD=49,500)	Final Apportionment
Andalusia	1,646	32.92	32	33.25	33
Barbary	6,936	138.72	138	140.12	140
Catalonia	154	3.08	3	3.11	3
Dover	2,091	41.82	41	42.24	42
Elyria	685	13.70	13	13.84	13
Faora	988	19.76	19	19.96	19
Totals:	12,500				250

there is a quota rule violation

Barbary should not receive more than 139 seats but ended up with 140 seats

4. The following tables calculate an apportionment using 50 seats has been calculated for you. Something has gone "wrong" between the two apportionments. Can you find the paradox? Which paradox is it? Explain how you know (explain in words or indicate in the table the information you used to arrive at your conclusion – why is this information relevant?). (10 points)

State	Population	Standard Quota	Lower Quota	Surplus Seats	Final Apportionment
Georgia	150	8.33	8		8
Hamlet	78	4.33	4		4
Ian	173	9.61	9	1	10
Jamie	204	11.33	11		11
Kirby	295	16.39	16	1	17
Totals:	900		48		50

State	Population	Standard Quota	Lower Quota	Surplus Seats	Final Apportionment
Georgia	150	8.25	8		8
Hamlet	78	4.29	4	1	5
Ian	181	9.96	9	1	10
Jamie	204	11.22	11		11
Kirby	296	16.28	16		16
Totals:	909		48		50

Hamlet had 0 population growth but Kirby had .3% population growth but lost a seat to Hamlet. This is the population paradox.

5. The following tables calculate an apportionment has been calculated for you. Something has gone "wrong" between the two apportionments. Can you find the paradox? Which paradox is it? Explain how you know. (10 points)

State	Population	Standard Quota (SD=1000)	Lower Quota	Surplus Seats	Final Apportionment
Llanos	10,450	10.45	10		10
Mercia	89,550	89.55	89	1	90
Totals:	100,000		99		100

State	Population	Standard Quota (SD=1002.38)	Lower Quota	Surplus Seats	Final Apportionment
Llanos	10,450	10.42	10	1	11
Mercia	89,550	89.34	89		89
Nubia	5,250	5.24	5		5
Totals:	105,250		104		105

This is the new states paradox. adding Nubia changed apportionment of old "states". when it shouldn't.

6. Suppose that you can buy a lottery ticket this week with a possible payout of $2 million, but to win you have to pick six correct numbers from 32 possibilities. Calculate the expectation and determine if the game is worth playing if you must pay $2 to play. (8 points)

$\frac{1}{32} \cdot \frac{1}{31} \cdot \frac{1}{30} \cdot \frac{1}{29} \cdot \frac{1}{28} \cdot \frac{1}{27} * 2{,}000{,}000 - \$2$ assumes order matters $= -1.997$

$\frac{1}{32P6} \cdot 2{,}000{,}000 - \$2 = .207$

if order doesn't matter, yes.

no, if it does.

7. Now suppose you are considering playing the same lottery as in question #6, but the payout this week is $15 million. Should you play under these circumstances? (8 points)

$\frac{1}{32P6} * 15{,}000{,}000 - \$2 = .022989\ldots$

yes, now playing is okay whether order matters or not

8. The following questions involve counting. Your answers should be whole numbers, not fractions. You may write your answers in permutation or combination notation, but you should also evaluate them for their numerical value. (6 points each)
   a. If I can use any number (0-9) and any letter (A-Z), how many six digit passwords are possible?

$(36)^6 = 2{,}176{,}782{,}336$

b. Suppose I tossed a fair coin 8 times. How many possible sequences of head and tails are possible?

$(2)^8 = 256$

c. Suppose I wanted to create a password for my computer that was random, but which I had some chance of pronouncing. I want it to contain a sequence of vowels (V) and consonants (C) in the following order: CVCCVCCV. How many ways can I create such a password?

$21 \cdot 5 \cdot 21 \cdot 21 \cdot 5 \cdot 21 \cdot 21 \cdot 5 = (21)^5 (5)^3 =$

$510,512,625$

d. How many ways can I choose either a 4 or a Jack from a standard deck of cards?

$4 + 4 = 8$

e. How many ways can I choose first a 4 and then a Jack if I pull two cards from a deck?

$4 * 4 = 16$

f. Suppose I need to choose a board of directors for my company and I need 15 people to sit on it. The pool of people I can choose from is 46. How many possible boards of directors could I create?

$46C15 = 5.117 E 11$

5.117×10^{11}

g. Suppose I am watching a horse-race and wish to bet on the top three winners. If there are 10 horses in the race, how many top three sequences are there for me to choose from?

$10P3 = 720$

9. Calculate the probability of the following situations. You should write the fractional value and then write the equivalent percentage rounded to one decimal place (after converting to a percent). If the value is less than 0.1%, go out to the first non-zero digit. (8 points each)

a. Suppose that I wish to draw a card from a standard deck. What is the probability it will be a spade?

$$\frac{13}{52} = \frac{1}{4} = 25\%$$

b. What is the probability that I will draw two spades in a row?

$$\frac{13}{52} \cdot \frac{12}{51} = 5.88\%$$

c. Suppose I have a bowl of marbles: 5 green ones, 2 blue ones, 7 white ones, and 3 red ones. What is the probability that I will draw a white one followed by a blue one?

$$\frac{7}{17} \cdot \frac{2}{16} = 5.15\%$$

d. If my bank is randomly choosing 8-letter passwords for my bank account, what is the probability that I will get the password described in problem #8c if only letters can be used?

$$\frac{21^5 5^3}{(26)^8} = 0.24\%$$

e. Suppose you have equal chances of having either a boy or a girl for each child. What is the probability you will have two boys and one girl if you have three kids?

$$(3C2)\frac{1}{8} = \frac{3}{8} = 37.5\%$$

f. If you roll two standard six-sided dice, what is the probability that the sum of the faces will be 5?

1+4, 2+3, 4+1, 3+2

$$\frac{4}{36} = 11.1\%$$

10. Compute the following values in your calculator: (5 points each)

a. ${}_{10}P_3$ 720

b. ${}_{11}C_6$ 462

c. 10! 3,628,800

11. Calculate the odds give the probabilities described. (8 points each)

a. The probability of an event is 3/11 8/11

for 3:8 / against 8:3

b. The probability of the event not happening is 0.6. [Hint: convert to a fraction!]

2/5 3/5

against 3:2 for 2:3

12. Calculate the probability of the event given the odds. (8 points each)

a. The odds for the event are 8:15.

$\frac{8}{23} = 34.78\%$

b. The odds against the event are 1:1.

$\frac{1}{2} = 50\%$

BONUS: Devise a probability question of your own (like #9's questions) and answer it. Harder problems are worth more if they are correct, so choose the hardest one you know how to do. Choose something besides the standard 6-sided die, coins, etc. for the most points.

answers will vary.