Name KEY

Math 285, Quiz #7, Spring 2012

Instructions: Show all work. You may use a calculator to check your work, but to receive full credit for any calculus, you must show the steps.

1. Solve the heat conduction problem $\alpha^2 u_{xx} = u_t$ in a rod of length 40 cm whose ends are maintained at 0°C for all t>0. Suppose that α=1. Find an expression for the temperature u(x,t) given that $u(x,0) = x, 0 < x < 40$.

$L = .4$ m

$u(0,t) = 0$

$u(L,t) = 0$

$u(x,0) = x$

$u(x,t) = X(x)T(t)$

$u_x = X'T$

$u_{xx} = X''T$

$u_t = XT'$

$X''T = XT' \Rightarrow \frac{X''}{X} = \frac{T'}{T} = -\lambda$

$X'' + \lambda X = 0 \qquad T' + \lambda T = 0$

$T' + \frac{n^2\pi^2}{L^2} T = 0$

$T = e^{\frac{-n^2\pi^2}{L^2} t}$

$\lambda = +\mu^2$

$X'' + \mu^2 X = 0$

$X = A\sin\mu x + B\cos\mu x$

$0 = 0 + B\cos\mu t \Rightarrow B = 0$

$0 = A\sin\mu L \qquad \mu L = n\pi \qquad \mu = \frac{n\pi}{L} \qquad \lambda = \frac{n^2\pi^2}{L^2} = \frac{n^2\pi^2}{(.4)^2}$

$$u(x,t) = \sum_{n=1}^{\infty} C_n \sin\frac{n\pi}{.4}x \, e^{\frac{-n^2\pi^2}{(.4)^2}t}$$

$$C_n = \frac{2}{.4}\int_0^{.4} x \sin\frac{n\pi}{.4}x \, dx$$

$u = x \qquad dv = \sin\frac{n\pi}{.4}dx$

$du = dx \qquad v = -\cos\frac{n\pi}{.4}x \cdot \frac{.4}{n\pi}$

$$-\frac{2}{.4}\int_0^{.4} \frac{.4}{n\pi}\cos\frac{n\pi}{.4}dx + -\frac{2}{.4}x\cos\frac{n\pi}{.4}\frac{.4}{n\pi}$$

$$\frac{-2}{n\pi}\sin\frac{n\pi}{.4}x \cdot \frac{.4}{n\pi}\Big|_0^{.4} - \frac{2}{n\pi}x\cos\frac{n\pi}{.4}\Big|_0^{.4}$$

$$\frac{.4}{n\pi}\left(\frac{-2}{n\pi}\right)\sin n\pi + \frac{2(.4)}{n^2\pi^2}\sin 0 - \frac{2}{n\pi}.4\cos n\pi + \frac{2}{n\pi}(0)\cos 0$$

$\frac{.8}{n\pi}\cos n\pi = \frac{.8}{n\pi}$ when n even

$= -\frac{.8}{n\pi}$ when n odd

$$\boxed{u(x,t) = \sum_{n=1}^{\infty} \frac{(-1)^n .8}{n\pi}\sin\frac{n\pi}{.4}x \, e^{-\frac{n^2\pi^2}{.4^2}t}}$$