Name KEY

Math 285, Quiz #3, Spring 2012

Instructions: Show all work.

1. Verify that the exact equation $dx + \left(\frac{x}{y} + siny\right) dy = 0$ can be solved by the use of an integrating factor. Find μ.

$N_x = \frac{1}{y}$

$M_y = 0$

$\frac{d\mu}{dx} = \frac{-N_x + M_y}{N}\mu = \frac{-\frac{1}{y}+0}{\frac{x}{y}+\sin y}\mu$ not one variable

$\frac{d\mu}{dy} = \frac{N_x - M_y}{M}\mu = \frac{\frac{1}{y}-0}{1}\mu = \frac{1}{y}\mu$ Separable

$\frac{d\mu}{dy} = \frac{1}{y}\mu \Rightarrow \int \frac{d\mu}{\mu} = \int \frac{dy}{y} \Rightarrow \ln\mu = \ln y \Rightarrow \boxed{\mu = y}$

2. Find the solution to the second order equation $2y'' - 3y' + y = 0, y(0) = 2, y'(0) = 1/2$. Determine the maximum value of the solution (if it exists) on t≥0, and find any points where y=0 on that same interval.

$2r^2 - 3r + 1 = 0$

$(2r-1)(r-1) = 0$

$r = \frac{1}{2}, r = 1$

$y_1 = e^{\frac{1}{2}t} \quad y_2 = e^t$

$y(t) = Ae^{\frac{1}{2}t} + Be^t$

$y(0) = 2 = A + B$

$y'(t) = \frac{1}{2}Ae^{\frac{1}{2}t} + Be^t$

$y'(0) = \frac{1}{2} = \frac{1}{2}A + B$

$(x-2)$

$2 = \cancel{A} + B$

$-1 = \cancel{-A} - 2B$

$1 = -B$

$B = -1$

$A - 1 = 2$

$A = 3$

$\boxed{y(t) = 3e^{\frac{1}{2}t} - e^t}$

b) $y'(t) = \frac{3}{2}e^{\frac{1}{2}t} - e^t = 0$

$\frac{\frac{3}{2}e^{\frac{1}{2}t}}{e^t} = \frac{e^t}{e^t} \Rightarrow e^{-\frac{1}{2}t} = \frac{2}{3}$ @MAX

$\frac{\ln(2/3)}{-\frac{1}{2}} = t \approx .8109$

$y(max) = 2.25$

c) $0 = 3e^{\frac{1}{2}t} - e^t$

$e^t = 3e^{\frac{1}{2}t}$

$\frac{1}{3} = e^{-\frac{1}{2}t}$

$\frac{\ln\frac{1}{3}}{-\frac{1}{2}} = t \approx 2.197$