

Name _____

KEY

Math 285, Quiz #2, Spring 2012

Instructions: Show all work. Give exact solutions. You are better off guessing on a step you aren't sure of and showing more of the process than stopping when you get stuck.

1. Solve the homogeneous equation $y' = -\frac{4x+3y}{2x+y}$.

$$y = vx$$

$$y' = v'x + v$$

$$v'x + v = \frac{-4x - 3vx}{2x + vx} = -\frac{4 - 3v}{2 + v} - v = -\frac{4 - 3v}{2 + v} - \frac{v(2 + v)}{2 + v}$$

$$v'x = \frac{-v^2 - 5v - 4}{2 + v} \Rightarrow \frac{dv(2 + v)}{-v^2 - 5v - 4} = \frac{1}{x} dx$$

$$\frac{dv(2 + v)}{-(v + 1)(v + 4)}$$

$$-\frac{2 + v}{(v + 1)(v + 4)} = \frac{A}{v + 1} + \frac{B}{v + 4}$$

$$A = \frac{2}{3}$$

$$B = \frac{4}{3}$$

$$\int \frac{A}{v + 1} + \frac{B}{v + 4} dv = \int \frac{1}{x} dx \Rightarrow A \ln|v + 1| + B \ln|v + 4| = \ln|x| + C$$

$$\ln|(v + 1)^{2/3} (v + 4)^{4/3}| = \ln|Cx|$$

$$(v + 1)^{2/3} (v + 4)^{4/3} = Cx$$

$$y = vx \Rightarrow v = \frac{y}{x}$$

$$\left(\frac{y}{x} + 1\right)^{2/3} (v + 4)^{4/3} = Cx$$

2. Simplify the Bernoulli equation $y' + xy = xy^{-1}$ until it is in linear form. (You don't need to solve it completely.)

$$y^n = y^{-1} \quad n = -1$$

$$y^{-n} (1 - n) = y' (1 - (-1)) = 2y$$

$$2y y' + 2xy^2 = 2x$$

$$z = y^2$$

$$z' = 2y y'$$

$$z' + 2xz = 2x$$