Homework #4 Math 285 Spring 2012

3.7

7. $w = 3$ lbs $\qquad$ 3 in $= \frac{1}{4}$ ft

$m = \frac{3}{32}$

$3 = k\,\frac{1}{4} \Rightarrow k = 12$

$y(0) = 1$

$y'(0) = -2$

$\frac{3}{32}y'' + 12y = 0$

$y'' + 128y = 0$

$r^2 + 128 = 0$

$r = \pm\sqrt{128}\,i = \pm 8\sqrt{2}\,i$

$y_1 = A\cos 8\sqrt{2}t$

$y_2 = B\sin 8\sqrt{2}t$

$y(t) = A\cos 8\sqrt{2}t + B\sin 8\sqrt{2}t$

$1 = A$

$y'(t) = -8\sqrt{2}A\sin 8\sqrt{2}t + 8\sqrt{2}B\cos 8\sqrt{2}t$

$\frac{-2}{8\sqrt{2}} = \cancel{8\sqrt{2}}B \qquad B = -\frac{\sqrt{2}}{4}$

$y(t) = \cos 8\sqrt{2}t - \frac{\sqrt{2}}{4}\sin 8\sqrt{2}t$

frequency $\omega = 8\sqrt{2}$

period $T = \frac{2\pi}{8\sqrt{2}}$

Amplitude $R = \sqrt{1^2 + \left(-\frac{\sqrt{2}}{4}\right)^2} = \sqrt{1 + \frac{2}{16}} = \sqrt{1 + \frac{1}{8}} = \sqrt{\frac{9}{8}} = \frac{3}{2\sqrt{2}}$

δ phase shift $= \tan^{-1}\left(\frac{-\sqrt{2}/4}{1}\right) = \tan^{-1}\left(-\sqrt{2}/4\right) = -.339837\ldots$

$0 = -10A + 4\sqrt{6}B$

$0 = -10(-2) + 4\sqrt{6}\,B$

$\frac{-\cancel{20}^{5}}{\cancel{4}\sqrt{6}} = \cancel{4\sqrt{6}}\,B \qquad B = \frac{5}{\sqrt{6}}$

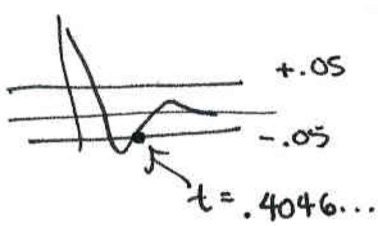
$y(t) = -2e^{-10t}\cos 4\sqrt{6}t - \frac{5}{\sqrt{6}}e^{-10t}\sin 4\sqrt{6}t$

undamped $\quad y'' + 196y = 0 \qquad \frac{4\sqrt{6}}{14} = .699854\ldots$

$r = \pm 14i$

natural frequency

$4\sqrt{6}$ quasi-frequency



17. $w = 8\text{ lbs} = F = k\left(\frac{1.5}{12}\right) \Rightarrow k = \frac{8\cdot 12}{1.5} = 64$

$m = \frac{8}{32} = \frac{1}{4}$

$\frac{1}{4}y'' + \gamma y' + 64y = 0$

$y'' + 4\gamma y' + 256y = 0$

$r = \frac{-4\gamma \pm \sqrt{(4\gamma)^2 - 4(256)}}{2}$ $\quad$ critical $\Rightarrow (4\gamma)^2 - 4(256) = 0$

$16\gamma^2 = 1024$

$\gamma^2 = 64$

$\gamma = \pm 8 \Rightarrow \gamma = 8$

γ in $\frac{\text{lbs}\cdot\text{sec}}{\text{ft.}}$

18. $C = .8 \times 10^{-6}$

$L = .2$

$$.2Q'' + RQ' + \frac{1}{.8\times10^{-6}}Q = 0$$

$$Q'' + 5RQ' + 6.25\times10^{6}\,Q = 0$$

$$\frac{-5R \pm \sqrt{25R^2 - 4(6.25\times10^6)}}{2} \Rightarrow$$

Critically damped

$$\frac{25R^2}{25} = \frac{2.5\times10^7}{25}$$

$$R^2 = 10^6$$

$$R = 10^3\,\Omega$$

29. $u'' + \frac{1}{4}u' + 2u = 0 \qquad u(0) = 0 \quad u'(0) = 2$

a.

$$r = \frac{-\frac{1}{4} \pm \sqrt{\frac{1}{16} - 8}}{2} = -\frac{1}{8} \pm \frac{\sqrt{127}}{8}i$$

$$u(t) = e^{-\frac{1}{8}t}\left(\cancel{A\cos\frac{\sqrt{127}}{8}t} + B\sin\frac{\sqrt{127}}{8}t\right)$$

$$0 = A$$

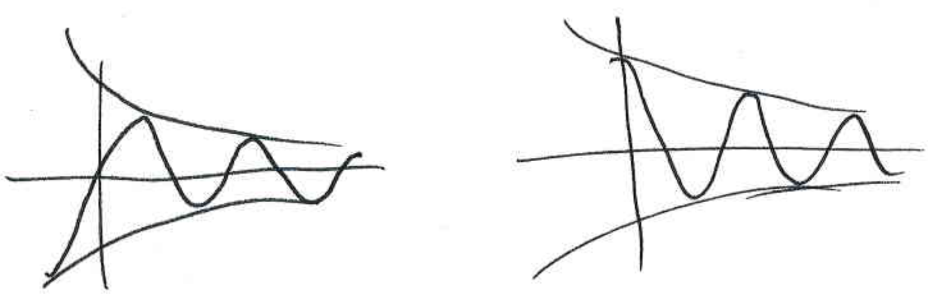
$$u'(t) = -\frac{1}{8}e^{-\frac{1}{8}t}\left(\cancel{A\cos\frac{\sqrt{127}}{8}t + B\sin\frac{\sqrt{127}}{8}t}\right) + e^{-\frac{1}{8}t}\left(\cancel{-\frac{\sqrt{127}}{8}A\sin\frac{\sqrt{127}}{8}t} + \frac{\sqrt{127}}{8}B\cos\frac{\sqrt{127}}{8}t\right)$$

$$2 = \cancel{-\frac{1}{8}(0)} + 1\left(\frac{\sqrt{127}}{8}B\right)$$

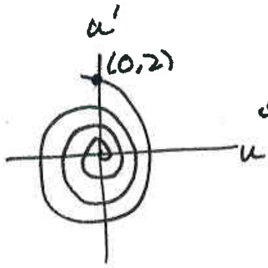
$$\frac{16}{\sqrt{127}} = B$$

$$u(t) = \frac{16}{\sqrt{127}}e^{-\frac{1}{8}t}\sin\frac{\sqrt{127}}{8}t$$

b.



c.



exponential spiral

you can graph this in your calculator by using the parametric settings

32. $F_s = -(ku + \varepsilon u^3)$

$\varepsilon > 0$ hardening

$\varepsilon < 0$ softening

these are appropriate because the value of ε increases or decreases the force as y increases

a) $mu'' + \gamma u' + ku + \varepsilon u^3 = 0 \qquad u(0) = 0 \quad u'(0) = 1$

$m = k = 1 \quad \gamma = 0$

$u'' + u + \varepsilon u^3 = 0 \implies u'' + u = -\varepsilon u^3$

$\varepsilon = 0 \quad u'' + u = 0$

$r^2 + 1 = 0$

$r = \pm i$

$u(t) = \cancel{A\cos t} + B\sin t$

$0 = A$

$u'(t) = B\cos t \qquad u(t) = \sin t$

$1 = B$

graphs for parts c-f are best obtained in Mathematica or other mathematical software

10 contd

$Y(t) = -2t\cos 8t$

$y(t) = A\cos 8t + B\sin 8t - 2t\cos 8t$

$y'(t) = -8A\sin 8t + 8B\cos 8t - 2\cos 8t + 16t\sin 8t$

$-1/4 = A$

$0 = 8B - 2$

$B = 1/4$

$y(t) = -1/4\cos 8t + \frac{1}{4}\sin 8t - 2t\cos 8t$

12. $K=3 \quad m=2 \quad \gamma=1$

$2y'' + y' + 3y = 3\cos 3t - 2\sin 3t$

$$\frac{-1 \pm \sqrt{1-4(2)(3)}}{4} = \frac{-1 \pm \sqrt{-23}}{4} = -\frac{1}{4} \pm \frac{\sqrt{23}}{4}i$$

$y_1 = e^{-1/4 t}\cos\frac{\sqrt{23}}{4}t$

$y_2 = e^{-1/4 t}\sin\frac{\sqrt{23}}{4}t$

transient part of solution

$Y(t) = A\cos 3t + B\sin 3t$

$Y'(t) = -3A\sin 3t + 3B\cos 3t$

$Y''(t) = -9A\cos 3t - 9B\sin 3t$

$-18A\cos 3t - 18B\sin 3t + 3B\cos 3t - 3A\sin 3t + 3A\cos 3t + 3B\sin 3t$

$\cos 3t\,(-18A + 3B + 3A) = 3\cos 3t$

$\sin 3t\,(-18B - \cancel{3A} + \cancel{3A}) = -2\sin 3t$

$-18B = -2 \qquad B = 1/9$

$$-15A + 3B = 3$$
$$-15A + 1/9 = 3$$
$$-15A = \frac{26}{9}$$
$$A = -\frac{26}{135}$$

$$Y(t) = \frac{-26}{135}\cos 3t + \frac{1}{9}\sin 3t$$ steady state

$$R = \sqrt{\left(\frac{-26}{135}\right)^2 + \left(\frac{1}{9}\right)^2} \approx .2223456\ldots$$

$$\omega = 3$$

$$\tan^{-1}\left(\frac{1/9}{-26/135}\right) = \tan^{-1}\left(\frac{-135}{26.9}\right) \approx -.523 = \delta$$

$$Y(t) = .222\cos(3t + .523)$$

13. $C = .25 \times 10^{-6}$ $\quad R = 5 \times 10^3$ $\quad L = 1$ $\quad Q(0) = 0$

$g(t) = 12$ $\quad Q'(0) = 0$

$$Q'' + 5\times10^3 Q' + \frac{1}{.25\times10^{-6}} Q = 0$$

$$Q'' + 5\times10^3 Q' + 4\times10^6 Q = 0$$

$$r = \frac{-5\times10^3 \pm \sqrt{(5\times10^3)^2 - 4(4\times10^6)}}{2} = \frac{-5\times10^3 \pm \overset{3\times10^3}{3000}}{2}$$

$$r_1 = \frac{-5+3}{2}\times10^3 = -10^3$$

$$r_2 = \frac{-5-3}{2}\times10^3 = -4\times10^3$$

$$Q_1 = e^{-10^3 t} \qquad Q_2 = e^{-4\times10^3 t}$$

13 cont'd

$g(t) = 12$

$Y(t) = B$

$Y'(t) = Y''(t) = 0$

$0 + 5\times10^3(0) + 4\times10^6(B) = 12$

$B = \frac{1}{3}\times10^{-6}$

$Q(t) = Ae^{-10^3 t} + Ce^{-4\times10^3 t} + \frac{1}{3}\times10^{-6}$

$0 = A + C + \frac{1}{3}\times10^{-6} \implies A + C = -\frac{1}{3}\times10^{-6}$

$Q'(t) = -10^3 Ae^{-10^3 t} - 4\times10^3 Ce^{-4\times10^3 t}$

$0 = -10^3 \cancel{A} - 4\times10^3 C$

$-\frac{1}{3}\times10^{-3} = \cancel{10^3 A} + 10^3 C$

$\frac{-\frac{1}{3}\times10^{-3}}{-3\times10^3} = \frac{-3\times10^3 C}{-3\times10^3}$

$C = \frac{1}{9}\times10^{-6}$

$A + \frac{1}{9}\times10^{-6} = -\frac{1}{3}\times10^{-6}$

$A = -\frac{4}{9}\times10^{-6}$

$Q = -\frac{4}{9}\times10^{-6}e^{-10^3 t} + \frac{1}{9}\times10^{-6}e^{-4\times10^3 t} + \frac{1}{3}\times10^{-6}$

$Q(.001) = 1.7\times10^{-7}$

$Q(.01) = 3.3\times10^{-7}$

limiting charge $= \frac{1}{3}\times10^{-6}$

18. $u'' + u = 3\cos\omega t \qquad u(0) = 0 \qquad u'(0) = 0$

$Y(t) = A\cos\omega t + B\sin\omega t \qquad r = \pm i$

$Y'(t) = -\omega A\sin\omega t + \omega B\cos\omega t \qquad y_1, y_2 = \cos t, \sin t$

$Y''(t) = -\omega^2 A\cos\omega t - \omega^2 B\sin\omega t$

$-\omega^2 A\cos\omega t - \omega^2 B\sin\omega t + A\cos\omega t + B\sin\omega t = 3\cos\omega t$

$-\omega^2 A + A = 3 \quad (\cos\omega t)$

$-\omega^2 B + B = 0 \quad (\sin\omega t)$

$B(-\omega^2 + 1) = 0$

$B = 0 \qquad A(-\omega^2 + 1) = 3 \Rightarrow A = \frac{3}{-\omega^2 + 1}$

$Y(t) = \left(\frac{-3}{\omega^2 - 1}\right)\cos\omega t$

$y(t) = C\cos t + D\sin t + \left(-\frac{3}{\omega^2 - 1}\right)\cos\omega t$

$0 = C + \left(\frac{-3}{\omega^2-1}\right) \Rightarrow C = \frac{3}{\omega^2 - 1}$

$y'(t) = -C\sin t + D\cos t + \frac{3\omega}{\omega^2-1}\sin\omega t$

$0 = D$

$y(t) = \frac{3}{\omega^2-1}\cos t - \left(\frac{3}{\omega^2-1}\right)\cos\omega t$

the graph gives beats. as ω approaches 1, the oscillations approach resonance & become larger & larger (in amplitude)

21. $u'' + .125u' + 4u = F(t) \qquad u(0) = 2 \qquad u'(0) = 0$

$= 0$

$r^2 + .125r + 4 = 0$

$r = \frac{-.125 \pm \sqrt{(.125)^2 - 4(4)}}{2} = -\frac{1}{16} \pm \frac{\sqrt{1023}\,i}{16}$

$y_1 = e^{-1/16 t}\cos\left(\frac{\sqrt{1023}}{16}t\right) \qquad y_2 = e^{-1/16 t}\sin\left(\frac{\sqrt{1023}}{16}t\right)$

21 cont'd

$F(t) = 3\cos(t/4)$

$Y(t) = A\cos(t/4) + B\sin(t/4)$

$Y'(t) = -\frac{1}{4}A\sin(t/4) + \frac{1}{4}B\cos(t/4)$

$Y''(t) = -\frac{1}{16}A\cos(t/4) - \frac{1}{16}B\sin(t/4)$

$\left(-\frac{1}{16}A + \frac{1}{8}\left(\frac{1}{4}B\right) + 4A\right)\cos(t/4) = 3\cos t/4$

$\left(-\frac{1}{16}B + \frac{1}{8}\left(-\frac{1}{4}A\right) + 4B\right)\sin(t/4) = 0$

$\frac{63}{16}A + \frac{1}{32}B = 3 \Rightarrow 126A + B = 96$

$\frac{63}{16}B - \frac{1}{32}A = 0 \Rightarrow 126B - A = 0 \Rightarrow A = 126B$

$126(126)B + B = 96$

$15877B = 96$

$B \approx .006$

$A \approx .762$

$Y(t) = .762\cos(t/4) + .006\sin(t/4)$

$y(t) = \left(C\cos\left(\frac{\sqrt{1023}}{16}t\right) + D\sin\left(\frac{\sqrt{1023}}{16}t\right)\right)e^{-1/16t} + .762\cos(t/4) + .006\sin(t/4)$

$y(0) = 2 = C + .762$

$C = 1.238$

$y'(t) = -1/16\, e^{-1/16t}\left(C\cos\left(\frac{\sqrt{1023}}{16}t\right) + D\sin\left(\frac{\sqrt{1023}}{16}t\right)\right) + e^{-1/16t}\left(-\frac{\sqrt{1023}}{16}C\sin\left(\frac{\sqrt{1023}}{16}t\right) + D\frac{\sqrt{1023}}{16}\cos\left(\frac{\sqrt{1023}}{16}t\right)\right) + \frac{-.762}{4}\sin(t/4) + \frac{.006}{4}\cos(t/4)$

$0 = -1/16(1)\left(\overset{1.238}{C}\right) + (1)\left(\frac{\sqrt{1023}}{16}D\right) + \frac{.006}{4}$

$.076 = \frac{\sqrt{1023}}{16}D \qquad D = -.038$

$y(t) = 1.238\, e^{-1/16t}\cos\left(\frac{\sqrt{1023}}{16}t\right) - .038\sin\left(\frac{\sqrt{1023}}{16}t\right)e^{-1/16t} + .762\cos(t/4) + .006\sin\frac{t}{4}$

21. cont'd

amplitude of forcing term is a 8:1 ratio (nearly) of the quasi-frequency/natural frequency of the system

22. $Y(t) = A\cos 2t + B\sin 2t$

$Y'(t) = -2A\sin 2t + 2B\cos 2t.$

$Y''(t) = -4A\cos 2t - 4B\sin 2t$

$(-4A + .125(2B) + A)\cos 2t = 3\cos 2t$

$-4A + \frac{1}{4}B + 4A = 3$

$+\frac{1}{4}B = 3$

$+B = 12$

$(-4B + .125(-2A) + 4B)\sin 2t = 0$

$-\frac{1}{4}A = 0 \Rightarrow A = 0$

$Y(t) = 12\sin 2t$

$y(t) = C\cos\left(\frac{\sqrt{1023}}{16}t\right)e^{-1/16t} + D\sin\left(\frac{\sqrt{1023}}{16}t\right)e^{-1/16t} + 12\sin 2t$

$2 = C$

$y'(t) = \left(-\frac{\sqrt{1023}}{8}\sin t\right)e^{-1/16t} - \frac{1}{8}e^{-1/16t}\cos\left(\frac{\sqrt{1023}}{16}t\right) + \frac{\sqrt{1023}}{16}D\cos\left(\frac{\sqrt{1023}}{16}t\right)e^{-1/16t}$

$- \frac{1}{16}D\sin\left(\frac{\sqrt{1023}}{16}t\right)e^{-1/16t} + 24\cos 2t$

$0 = 0 - \frac{1}{8} + \frac{\sqrt{1023}}{16}D + 24$

$-23.875 = \frac{\sqrt{1023}}{16}D$

$\frac{382}{\sqrt{1023}} = D$

$y(t) = 2\cos\left(\frac{\sqrt{1023}}{16}t\right)e^{-1/16t} + \frac{382}{\sqrt{1023}}\sin\left(\frac{\sqrt{1023}}{16}t\right)e^{-1/16t} + 12\sin 2t$

22 cont'd.

transient dies slowly & while it survives its in near perfect resonance w/ forcing functions.
depending on tolerance of the mechanical system, it could break before the transient dies.

5.1

#2 $\sum_{n=0}^{\infty} \frac{n}{2^n} x^n$

$$\lim_{n\to\infty}\left|\frac{(n+1)x^{\cancel{n+1}}}{2^{\cancel{n+1}}}\cdot\frac{\cancel{2^n}}{n\,\cancel{x^n}}\right| = \cancel{\lim_{n\to\infty}\left|\frac{n+1}{n}\right|}^{1}\lim_{n\to\infty}\left|\frac{x}{2}\right| < 1$$

$$-1 < \frac{x}{2} < 1$$

$$-2 < x < 2$$

Radius of convergence = 2

#8. $\sum_{n=1}^{\infty} \frac{n!\,x^n}{n^n}$

$$\lim_{n\to\infty}\left|\frac{(n+1)!\,x^{n+1}}{\underset{(n+1)^n(n+1)}{(n+1)^{n+1}}}\cdot\frac{n^n}{n!\,x^n}\right| = \lim_{n\to\infty}\left|\frac{(n+1)^n}{n^n}\right|^{-1}\cdot\lim_{n\to\infty}|x| < 1$$

$$\lim_{n\to\infty}\left|\frac{n+1}{n}\right|^{-n} = \lim\left(\frac{1}{n}+1\right)^{-n} = \lim_{n\to\infty}\left[\left(1+\frac{1}{n}\right)^n\right]^{-1} = e^{-1} = \frac{1}{e}$$

by memory or can prove by L'Hopital's together w/ log rules

$$\frac{1}{e}\lim_{n\to\infty}|x| < 1 \quad\Rightarrow\quad \left|\frac{1}{e}\cdot x\right| < 1$$

$$-1 < \frac{x}{e} < 1$$

$$-e < x < e$$

radius of convergence = e

12. $f(x) = x^2 \qquad x_0 = -1$

$f(-1) = 1$

$f'(x) = 2x \qquad f'(-1) = -2$

$f''(x) = 2 \qquad f''(-1) = 2$

$f'''(x) = 0$ etc.

$$1 - 2(x+1) + \frac{2}{2}(x+1)^2 = 1 - 2(x+1) + (x+1)^2$$

radius of convergence $= \infty$

taylor series terminates so this is exact

13. $\ln(x) = f(x) \qquad x_0 = 1 \qquad f(1) = 0$

$f'(x) = \frac{1}{x} \qquad f'(1) = 1$

$f''(x) = -\frac{1}{x^2} \qquad f''(1) = -1$

$f'''(x) = \frac{+2}{x^3} \qquad f'''(1) = 2$

$f^{IV}(x) = \frac{-6}{x^4} \qquad f^{IV}(1) = -6$

$\vdots$

$$1 - 1(x-1) + \frac{2}{2}(x-1)^2 + \frac{-6}{3!}(x-1)^3 + \cdots$$

$$= \sum_{n=0}^{\infty} (-1)^n (x-1)^n$$

$$\lim_{n\to\infty} \left| \frac{(x-1)^{n+1}}{(x-1)^n} \right| < 1 \Rightarrow \lim_{n\to\infty} |x-1| < 1$$

$-1 < x-1 < 1$ radius of convergence is 1

16. $\dfrac{1}{1-x}$ $\quad x_0 = 2$

$$f(x) = \frac{1}{1-x} = \frac{1}{1-(x-2)-2} = \frac{1}{-1-(x-2)} = \frac{-1}{1+(x-2)}$$

$$= \sum_{n=0}^{\infty} -1(-1)^n(x-2)^n = \sum_{n=0}^{\infty} (-1)^{n+1}(x-2)^n$$

$$\lim_{n\to\infty} \left|\frac{(x-2)^{n+1}}{(x-2)^n}\right| < 1 \Rightarrow |x-2| < 1$$

$$-1 < x-2 < 1$$ radius of convergence is 1

25. $$\sum_{m=2}^{\infty} m(m-1)a_m x^{m-2} + x\sum_{k=1}^{\infty} k a_k x^{k-1}$$

$$\sum_{m=0}^{\infty} (m+2)(m+1)a_{m+2}x^m + \sum_{k=1}^{\infty} k a_k x^k$$

$$\sum_{m=1}^{\infty} (m+2)(m+1)a_{m+2}x^m + 2(1)a_0 + \sum_{k=1}^{\infty} k a_k x^k$$

$$\sum_{n=1}^{\infty}\left((n+2)(n+1)a_{n+2}x^n + n a_n x^n\right) + 2a_0$$

$$\sum_{n=1}^{\infty}\left[(n+2)(n+1)a_{n+2} + n a_n\right]x^n + 2a_0$$

5.2.

#4 $y'' + k^2x^2y = 0 \qquad x_0 = 0$

$$y = \sum_{n=0}^{\infty} a_n x^n$$

$$y' = \sum_{n=1}^{\infty} a_n n x^{n-1}$$

$$y'' = \sum_{n=2}^{\infty} a_n n(n-1)x^{n-2} = \sum_{n=0}^{\infty} a_{n+2}(n+2)(n+1)x^n$$

4 cont'd

$$\sum_{n=0}^{\infty} a_{n+2}(n+2)(n+1)x^n + k^2x^2\sum_{n=0}^{\infty} a_n x^n = 0$$

$$\sum_{n=0}^{\infty} a_{n+2}(n+2)(n+1)x^n + k^2\sum_{n=0}^{\infty} a_n x^{n+2} = 0$$

$$\sum_{n=2}^{\infty} a_n(n)(n-1)x^n + k^2\sum_{n=2}^{\infty} a_{n-2}x^n + k^2(a_0 + a_1 x)$$

$$\sum_{n=2}^{\infty}\left[a_n n(n-1) + k^2 a_{n-2}\right]x^n + k^2 a_0 + k^2 a_1 x = 0$$

$a_0 = 0$

$a_1 = 0$

$a_n(n)(n-1) + k^2(a_{n-2}) = 0$

$\dfrac{a_n(n)(n-1)}{-k^2} = \cancel{-k^2}\, a_{n-2}$

all terms $= 0$

5. $(1-x)y'' + y = 0$

$$(1-x)\sum_{n=0} a_{n+2}(n+2)(n+1)x^n + \sum_{n=0}^{\infty} a_n x^n = 0$$

$$\sum_{n=0}^{\infty} a_{n+2}(n+2)(n+1)x^n - \sum_{n=0}^{\infty} a_{n+2}(n+2)(n+1)x^{n+1} + \sum_{n=0}^{\infty} a_n x^n = 0$$

$$\sum_{n=1}^{\infty} a_{n+2}(n+2)(n+1)x^n + a_0(2) - \sum_{n=1}^{\infty} a_{n+1}(n+1)(n)x^n + \sum_{n=1}^{\infty} a_n x^n + a_0 = 0$$

$$\sum_{n=1}^{\infty}\left[a_{n+2}(n+2)(n+1) - a_{n+1}(n+1)(n) + a_n\right]x^n + a_2(2) + a_0 = 0$$

$a_{n+2}(n+2)(n+1) - a_{n+1}(n+1)(n) + a_n = 0$

$a_2 = \frac{a_0}{2}$

5 cont'd

$$a_2 = -\frac{a_0}{2} \qquad a_{n+2} = \frac{a_{n+1}(n+1)(n) - a_n}{(n+2)(n+1)}$$

$a_1 = a_1$

$n=1$

$$a_3 = \frac{a_2(2)(1) - a_1}{3 \cdot 2} = \frac{-a_0 - a_1}{6}$$

$$a_4 = \frac{a_3(3)(2) - a_2}{4 \cdot 3} = \frac{\frac{-a_0 - a_1}{6} + \frac{a_0}{2}}{12} = \frac{\frac{1}{3}a_0 - \frac{1}{6}a_1}{12} = \frac{1}{36}a_0 - \frac{1}{72}a_1$$

etc.

Choose $a_0 = 0$

$$\Rightarrow a_1\left(x - \frac{x^3}{6} - \frac{x^4}{12} - \frac{x^5}{24} \cdots\right)$$

Choose $a_1 = 0$

$$a_0\left(1 - \frac{1}{2}x^2 - \frac{1}{6}x^3 + \frac{1}{24}x^4 + \cdots\right)$$

9. $(1+x^2)y'' - 4xy' + 6y = 0$

$$(1+x^2)\sum_{n=2}^{\infty} a_n(n)(n-1)x^{n-2} - 4x\sum_{n=1}^{\infty} a_n n x^{n-1} + 6\sum a_n x^n = 0$$

$$\sum_{n=2}^{\infty} a_n(n)(n-1)x^{n-2} + \sum_{n=2}^{\infty} a_n(n)(n-1)x^n - \sum_{n=1}^{\infty} 4a_n n x^n + \sum_{n=0}^{\infty} 6a_n x^n = 0$$

$$\sum_{n=0}^{\infty} a_{n+2}(n+2)(n+1)x^n + \sum_{n=2}^{\infty} a_n(n)(n-1)x^n - \sum_{n=1}^{\infty} 4a_n n x^n + \sum_{n=0} 6a_n x^n = 0$$

$$a_2(2)(1) + a_3(3)(2)x + \sum_{n=2}^{\infty} a_{n+2}(n+2)(n+1)x^n + \sum_{n=2}^{\infty} a_n n(n-1)x^n - \sum_{n=2}^{\infty} 4a_n n x^n - \cancel{4a_1(0)}$$
$$+ \sum_{n=2}^{\infty} 6a_n x^n + 6a_0 + 6a_1 x = 0$$

$$\sum_{n=2}^{\infty}\left[a_{n+2}(n+2)(n+1) + a_n n(n-1) - 4a_n n + 6a_n\right]x^n + 2a_2 + 6a_3 x + 6a_0 + 6a_1 x = 0$$

9 contd

$a_{n+2}(n+2)(n+1) + a_n[n(n-1) - 4n + 6] = 0$

$2a_2 + 6a_0 = 0 \Rightarrow a_2 + 3a_0 = 0 \quad a_2 = -3a_0$

$6a_3 + 6a_1 = 0 \Rightarrow a_3 = -a_1$

$$a_{n+2} = \frac{-a_n[n(n-1) - 4n + 6]}{(n+2)(n+1)} = \quad n^2 - n - 4n + 6 = n^2 - 5n + 6 = (n-2)(n-3)$$

$$= \frac{-a_n(n-2)(n-3)}{(n+2)(n+1)}$$

21. $y'' + 2xy' + \lambda y = 0$

$$y = \sum_{n=0}^{\infty} a_n x^n$$

$$\sum_{n=0}^{\infty} a_{n+2}(n+2)(n+1)x^n + 2x\sum_{n=1}^{\infty} a_n n x^{n-1} + \lambda \sum_{n=0}^{\infty} a_n x^n = 0$$

$$\sum_{n=1}^{\infty} 2a_n n x^n$$

$$\sum_{n=1}^{\infty} a_{n+2}(n+2)(n+1)x^n + a_2(2) + \sum_{n=1}^{\infty} 2a_n n x^n + \lambda \sum_{n=1}^{\infty} a_n x^n + \lambda a_0 = 0$$

$$\sum_{n=1}^{\infty} [a_{n+2}(n+2)(n+1) + 2a_n n + \lambda a_n] x^n + 2a_2 + \lambda a_0 = 0$$

$2a_2 + \lambda a_0 = 0$

$2a_2 = -\lambda a_0 \Rightarrow a_2 = \frac{-\lambda a_0}{2}$

$a_{n+2}(n+2)(n+1) + 2a_n n + \lambda a_n = 0$

$$a_{n+2} = \frac{-a_n(2n+\lambda)}{(n+2)(n+1)}$$

21 cont'd