Name KEY

Math 268, Quiz #4, Spring 2012

Instructions: Show all work. Calculators should be used to check, not to perform work. Justify your answers.

1. Consider the set of polynomials $\{1, t+t^2, 3-t^2+2t^3, -4t-t^3\}$. Does this set form a basis of $\boldsymbol{P}_3$? In other words, does the set span $\boldsymbol{P}_3$ and is the set linearly independent? Does it satisfy the definition of a subspace? [Hint: treat the coefficients of each term as entries in the 4x1 vector.]

$$\begin{bmatrix} - \\ - \\ - \\ - \end{bmatrix}\begin{matrix} t^3 \\ t^2 \\ t \\ 1 \end{matrix} \rightarrow \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ -1 \\ 3 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ -4 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 2 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & -1 & -4 \\ 1 & 0 & 3 & 0 \end{bmatrix} \text{ rref} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

yes it is a basis since it spans $\mathbb{P}_3$ & is linearly independent.

yes, it is a subspace.

2. Find the kernel and the column space of the matrix $A = \begin{bmatrix} 4 & -5 & 2 & 3 & 0 \\ 1 & 1 & 0 & 1 & 0 \end{bmatrix}$.

$$\text{rref} \Rightarrow \begin{bmatrix} 1 & 0 & 2/9 & 8/9 & 0 \\ 0 & 1 & -2/9 & 1/9 & 0 \end{bmatrix}$$

free variables x_3, x_4, x_5

$x_1 = -2/9\, x_3 - 8/9\, x_4$

$x_2 = 2/9\, x_3 - 1/9\, x_4$

$x_3 = x_3$

$x_4 = x_4$

$x_5 = x_5$

$$\Rightarrow \vec{x} = \begin{bmatrix} -2/9 \\ 2/9 \\ 1 \\ 0 \\ 0 \end{bmatrix} x_3 + \begin{bmatrix} -8/9 \\ -1/9 \\ 0 \\ 1 \\ 0 \end{bmatrix} x_4 + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} x_5$$

Nul A =

$\Rightarrow$ kernel: $\left\{ \begin{bmatrix} -2/9 \\ 2/9 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -8/9 \\ -1/9 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$