Name KEY

Math 268, Quiz #2.5, Spring 2012

1. Consider the transformation $T: P_n \to \mathbb{R}$ such that $T(f) = \int_0^t f(x)dx$. If f(x) is any polynomial in P_n, use the definition of a linear transformation to show that T is linear.

3 conditions to check: 1) $T(u+v) = T(u) + T(v)$
2) $T(cu) = cT(u)$
3) $T(0) = 0$

$f = u,\ g = v$

1) $\int_0^t (f(x) + g(x))dx = \int_0^t f(x)dx + \int_0^t g(x)dx$ ✓

2) $\int_0^t kf(x)dx = k\int_0^t f(x)dx$ ✓

3) $\int_0^t 0\,dx = 0$ ✓

all by properties of integrals

2. Compare Problem #1 to the following: Consider the transformation $T: \mathbb{R}^3 \to \mathbb{R}^3$ such that $T(\vec{x}) = A\vec{x}$. If $\vec{x}$ is any vector in $\mathbb{R}^3$, use the definition of a linear transformation to show that T is linear.

$u = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad v = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$

$T(u+v) = A(u+v)$ by matrix multiplication $= Au + Av$ ✓

$T(cu) = A(cu)$ " " " $= cAu$ ✓

$T(0) = A\vec{0} = \vec{0}$ ✓

by properties of matrix multiplication
you don't even need to know what matrix you are using