Homework #5 key Math 268 Spring 2012

1\. a.

$$\begin{bmatrix} .70 & .10 & .05 \\ .20 & .50 & .20 \\ .10 & .40 & .75 \end{bmatrix} = A$$

b. $\begin{bmatrix} .80 \\ .10 \\ .10 \end{bmatrix} = \vec{x}_0$ $A\vec{x}_0 = \begin{bmatrix} .575 \\ .23 \\ .195 \end{bmatrix}$

57.5% receive an A
23% receive a B
19.5% receive less than a B.

c. $\begin{bmatrix} .183673 \\ .285714 \\ .530612 \end{bmatrix}$

18.37% receive an A
28.57% receive a B
53.06% receive a C or less

2\.
a. $\begin{bmatrix} .1 & .7 \\ .9 & .3 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -.9 & .7 \\ .9 & -.7 \end{bmatrix}$

$$-.9x_1 + .7x_2 = 0$$

$$x_1 = \frac{-.7x_2}{-.9} \Rightarrow x_1 = \frac{7}{9}x_2 \qquad \begin{bmatrix} 7 \\ 9 \end{bmatrix} \Rightarrow \begin{bmatrix} 7/16 \\ 9/16 \end{bmatrix}$$

b. $\begin{bmatrix} .6 & .2 & .04 \\ .25 & .5 & .01 \\ .15 & .3 & .95 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -.4 & .2 & .04 \\ .25 & -.5 & .01 \\ .15 & .3 & -.05 \end{bmatrix}$ rref $\Rightarrow$

$$\begin{bmatrix} 1 & 0 & -.146\bar{6} \\ 0 & 1 & -.09\bar{3} \\ 0 & 0 & 0 \end{bmatrix}$$

$x_1 = .146\bar{6}\,x_3$ $x_1 = 11/75\,x_3$
$x_2 = +.093\bar{3}\,x_3$ $x_2 = 7/75\,x_3$
$x_3 = x_3$ $x_3 = x_3$

$$\begin{bmatrix} 11 \\ 7 \\ 75 \end{bmatrix} \Rightarrow \begin{bmatrix} 11/93 \\ 7/93 \\ 75/93 \end{bmatrix}$$

3. a. true

b. true

c. true

d. false, if the characteristic equation has repeated roots, it's possible for a single eigenvalue to be associated w/ multiple eigenvectors

e. true

f. false. only if the matrix is triangular

g. false an nxn matrix can have no more than n eigenvalues

h. false. see f.

i. false. only one of the 3 operations doesn't change the determinant

j. false $\lambda = -5$

4. a. $\begin{bmatrix} 8-\lambda & 2 \\ 3 & 3-\lambda \end{bmatrix}$ $(8-\lambda)(3-\lambda)-6 = 24-11\lambda+\lambda^2-6 =$

$18-11\lambda+\lambda^2=0$

$(\lambda-9)(\lambda-2)=0$ $\lambda_1=9,\ \lambda_2=2$

$\begin{bmatrix} 8-9 & 2 \\ 3 & 3-9 \end{bmatrix} = \begin{bmatrix} -1 & 2 \\ 3 & -6 \end{bmatrix}$ $-x_1+2x_2=0$ $v_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

$x_1 = 2x_2$

$\begin{bmatrix} 8-2 & 2 \\ 3 & 3-2 \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 3 & 1 \end{bmatrix}$ $6x_1+2x_2=0$

$x_1 = -\frac{2x_2}{6} = -\frac{1}{3}x_2$ $v_2 \begin{bmatrix} 1 \\ -3 \end{bmatrix}$

b. $\begin{bmatrix} -4-\lambda & 3 \\ 2 & 1-\lambda \end{bmatrix}$ $(-4-\lambda)(1-\lambda)-6 = -4+4\lambda-\lambda+\lambda^2-6 =$

$\lambda^2+3\lambda-10=0$

$(\lambda+5)(\lambda-2)=0$ $\lambda_1=-5,\ \lambda_2=2$

$\begin{bmatrix} -4-(-5) & 3 \\ 2 & 1-(-5) \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix}$ $x_1+3x_2=0$ $v_1 = \begin{bmatrix} -3 \\ 1 \end{bmatrix}$

$x_1 = -3x_2$

4b cont'd

$$\begin{bmatrix} -4-2 & 3 \\ 2 & 1-2 \end{bmatrix} = \begin{bmatrix} -6 & 3 \\ 2 & -1 \end{bmatrix} \qquad 2x_1 - x_2 = 0 \qquad 2x_1 = x_2$$

$$x_1 = \tfrac{1}{2}x_2 \qquad v_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

4c. $\begin{vmatrix} 3-\lambda & 0 & 0 \\ 2 & 1-\lambda & 4 \\ 1 & 0 & 4-\lambda \end{vmatrix} = (3-\lambda)\begin{vmatrix} 1-\lambda & 4 \\ 0 & 4-\lambda \end{vmatrix} = (3-\lambda)(1-\lambda)(4-\lambda) = 0$

$\lambda_1 = 3,\ \lambda_2 = 1,\ \lambda_3 = 4$

$$\begin{bmatrix} 0 & 0 & 0 \\ 2 & 1-3 & 4 \\ 1 & 0 & 4-3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 2 & -2 & 4 \\ 1 & 0 & 1 \end{bmatrix} \text{ rref} \Rightarrow \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$x_1 = -x_3 \qquad x_2 = x_3 \qquad x_3 = x_3 \qquad v_1 = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 3-1 & 0 & 0 \\ 2 & 1-1 & 4 \\ 1 & 0 & 4-1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ 2 & 0 & 4 \\ 1 & 0 & 3 \end{bmatrix} \text{ rref} \Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \qquad x_1 = 0,\ x_2 = x_2,\ x_3 = 0 \qquad v_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 3-4 & 0 & 0 \\ 2 & 1-4 & 4 \\ 1 & 0 & 4-4 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 2 & -3 & 4 \\ 1 & 0 & 0 \end{bmatrix} \text{ rref} \Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -4/3 \\ 0 & 0 & 0 \end{bmatrix}$$

$$x_1 = 0 \qquad x_2 = 4/3\, x_3 \qquad x_3 = x_3 \qquad v_3 = \begin{bmatrix} 0 \\ 4 \\ 3 \end{bmatrix}$$

d. $\det \begin{bmatrix} 5-\lambda & 5 & 0 & 2 \\ 0 & 2-\lambda & -3 & 6 \\ 0 & 0 & 3-\lambda & -2 \\ 0 & 0 & 0 & 5-\lambda \end{bmatrix}$ $\quad (5-\lambda)(2-\lambda)(3-\lambda)(5-\lambda) = 0$

$5 = \lambda_1 \quad 2 = \lambda_2 \quad 3 = \lambda_3 \qquad$ 5 is repeated

$$\begin{bmatrix} 0 & 5 & 0 & 2 \\ 0 & 2-5 & -3 & 6 \\ 0 & 0 & 3-5 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 5 & 0 & 2 \\ 0 & -3 & -3 & 6 \\ 0 & 0 & -2 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \text{ rref} \Rightarrow \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$x_1 = x_1 \qquad x_2 = 0 \qquad x_3 = 0 \qquad x_4 = 0 \qquad v_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

4d. $\begin{bmatrix} 5-2 & 5 & 0 & 2 \\ 0 & 2-2 & -3 & 6 \\ 0 & 0 & 3-2 & -2 \\ 0 & 0 & 0 & 5-2 \end{bmatrix} = \begin{bmatrix} 3 & 5 & 0 & 2 \\ 0 & 0 & -3 & 6 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 3 \end{bmatrix} \text{ rref} = \begin{bmatrix} 1 & 5/3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$x_1 = -5/3\, x_2$
$x_2 = x_2$
$x_3 = 0$
$x_4 = 0$

$v_1 = \begin{bmatrix} -5 \\ 3 \\ 0 \\ 0 \end{bmatrix}$

$\begin{bmatrix} 5-3 & 5 & 0 & 2 \\ 0 & 2-3 & -3 & 6 \\ 0 & 0 & 3-3 & -2 \\ 0 & 0 & 0 & 5-3 \end{bmatrix} = \begin{bmatrix} 2 & 5 & 0 & 2 \\ 0 & -1 & -3 & 6 \\ 0 & 0 & 0 & -2 \\ 0 & 0 & 0 & 2 \end{bmatrix} \text{ rref} = \begin{bmatrix} 1 & 0 & -15/2 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$x_1 = 15/2\, x_3$
$x_2 = -3x_3$
$x_3 = x_3$
$x_4 = 0$

$v_3 = \begin{bmatrix} 15 \\ -6 \\ 2 \\ 0 \end{bmatrix}$

4e. $\begin{bmatrix} 1-\lambda & -2 \\ 1 & 3-\lambda \end{bmatrix}$ $\quad (1-\lambda)(3-\lambda)+2=0 \quad 3-3\lambda-\lambda+\lambda^2+2=0$

$5-4\lambda+\lambda^2=0$

$\dfrac{4 \pm \sqrt{16-20}}{2} = \dfrac{4 \pm 2i}{2} = 2 \pm i = \lambda_1, \lambda_2$

$\begin{bmatrix} 1-(2+i) & -2 \\ 1 & 3-(2+i) \end{bmatrix} = \begin{bmatrix} -1-i & -2 \\ 1 & 1-i \end{bmatrix}$

$x_1 = -(1-i)x_2$
$x_1 = (-1+i)x_2$

$v_1 = \begin{bmatrix} -1+i \\ 1 \end{bmatrix}$
$= \begin{bmatrix} -1 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} i$

$v_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix} + \begin{bmatrix} -1 \\ 0 \end{bmatrix} i$

4f. $\begin{bmatrix} -\lambda & 5 \\ -2 & 2-\lambda \end{bmatrix}$ $\quad -\lambda(2-\lambda)+10 = -2\lambda+\lambda^2+10 = \lambda^2-2\lambda+10=0$

$$\frac{2 \pm \sqrt{4-40}}{2} = \frac{2 \pm 6i}{2} = 1 \pm 3i = \lambda_1 \lambda_2$$

$$\begin{bmatrix} -1-3i & 5 \\ -2 & 2-(1+3i) \end{bmatrix} = \begin{bmatrix} -1-3i & 5 \\ -2 & -1-3i \end{bmatrix}$$

$$-2x_1 = (1+3i)x_2$$

$$x_1 = \left(\frac{-1}{2} - \frac{3}{2}i\right)x_2$$

$$v_1 = \begin{bmatrix} -1-3i \\ 2 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix} - \begin{bmatrix} 3 \\ 0 \end{bmatrix} i$$

$$v_2 = \begin{bmatrix} -1 \\ 2 \end{bmatrix} + \begin{bmatrix} 3 \\ 0 \end{bmatrix} i$$

4g. $\begin{bmatrix} -3-\lambda & -8 \\ 4 & 5-\lambda \end{bmatrix}$ $\quad (-3-\lambda)(5-\lambda)+32$

$$-15+3\lambda-5\lambda+\lambda^2+32 = \lambda^2-2\lambda+17=0$$

$$\frac{-2 \pm \sqrt{4-68}}{2} = \frac{-2 \pm 8i}{2} = -1 \pm 4i = \lambda_1, \lambda_2$$

$$\begin{bmatrix} -3-(-1+4i) & -8 \\ 4 & 5-(-1+4i) \end{bmatrix} = \begin{bmatrix} -2-4i & -8 \\ 4 & 6-4i \end{bmatrix}$$

$$4x_1 + (6-4i)x_2 = 0$$

$$x_1 = \frac{(-6+4i)}{4}x_2$$

$$x_1 = \left(-\frac{3}{2}+i\right)x_2$$

$$v_1 = \begin{bmatrix} -3+2i \\ 2 \end{bmatrix} = \begin{bmatrix} -3 \\ 2 \end{bmatrix} + \begin{bmatrix} 2 \\ 0 \end{bmatrix} i$$

$$v_2 = \begin{bmatrix} -3 \\ 2 \end{bmatrix} + \begin{bmatrix} -2 \\ 0 \end{bmatrix} i$$

5e. $\begin{bmatrix} 1 & -2 \\ 1 & 3 \end{bmatrix} = A$

$P = \begin{bmatrix} -1 & 1 \\ 1 & 0 \end{bmatrix}$ $C = \begin{bmatrix} 2 & 1 \\ -1 & 2 \end{bmatrix}$ $PCP^{-1} = \begin{bmatrix} 1 & -2 \\ 1 & 3 \end{bmatrix}$

5f. $A = \begin{bmatrix} 0 & -2 \\ 5 & 2 \end{bmatrix}$ $P = \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}$ $C = \begin{bmatrix} 1 & 3 \\ -3 & 1 \end{bmatrix}$

$PCP^{-1} = \begin{bmatrix} 0 & -.5 \\ 2 & 2 \end{bmatrix}$

5g. $A = \begin{bmatrix} -3 & -8 \\ 4 & 5 \end{bmatrix}$ $P = \begin{bmatrix} -3 & -2 \\ 2 & 0 \end{bmatrix}$ $C = \begin{bmatrix} -1 & 4 \\ -4 & -1 \end{bmatrix}$

$PCP^{-1} =$

$p = .325$

6. $\begin{bmatrix} .4 & .3 \\ -.325 & 1.2 \end{bmatrix} \to \begin{bmatrix} .4-\lambda & .3 \\ -.325 & 1.2-\lambda \end{bmatrix}$ $(.4-\lambda)(1.2-\lambda) + .325 * .3 = 0$

$\lambda = .55$ $\lambda = 1.05$

$\begin{bmatrix} .4-1.05 & .3 \\ -.325 & 1.2-1.05 \end{bmatrix} = \begin{bmatrix} -.65 & .3 \\ -.325 & -.15 \end{bmatrix}$ $-.65x_1 + .3x_2 = 0$

$x_1 = \frac{-.3x_2}{-.65}$ $x_1 = \frac{6}{13}x_2$

$\underline{v}_2 = \begin{bmatrix} 6 \\ 13 \end{bmatrix}$

Origin is a saddle point

x_k approaches $\begin{bmatrix} 6 \\ 13 \end{bmatrix}$ in long term

$p = .104$

$\begin{bmatrix} .4 & .3 \\ -.104 & 1.2 \end{bmatrix} \to \begin{bmatrix} .4-\lambda & .3 \\ -.104 & 1.2-\lambda \end{bmatrix}$ $(.4-\lambda)(1.2-\lambda) + .104 * .3 = 0$

$\lambda = .44..$ $\lambda = 1.15$

behaves similar to above

6 cont'd

$\begin{bmatrix} .4 & .3 \\ -.855 & 1.2 \end{bmatrix} \Rightarrow \begin{bmatrix} .4-\lambda & .3 \\ -.855 & 1.2-\lambda \end{bmatrix}$ $(.4-\lambda)(1.2-\lambda) + .855 * .3 = 0$

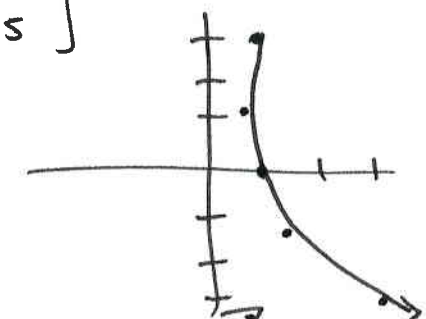
no real eigenvalues

7. a. $\begin{bmatrix} 1.7-\lambda & -.3 \\ -1.2 & .8-\lambda \end{bmatrix}$ $(1.7-\lambda)(.8-\lambda) - 1.2 * .3 = 0$

$\lambda = .5 \quad \lambda = 2$

saddle point

$\begin{bmatrix} 5 \\ 15 \end{bmatrix}, \begin{bmatrix} 4 \\ 6 \end{bmatrix}, \begin{bmatrix} 5 \\ 0 \end{bmatrix}, \begin{bmatrix} 8.5 \\ -6 \end{bmatrix}, \begin{bmatrix} 16.25 \\ -15 \end{bmatrix}, \begin{bmatrix} 32.125 \\ -31.5 \end{bmatrix}$

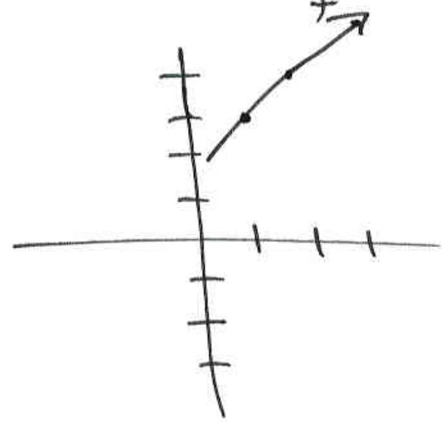


7b. $\begin{bmatrix} .5-\lambda & .6 \\ -.3 & 1.4-\lambda \end{bmatrix}$ $(.5-\lambda)(1.4-\lambda) + .3 * .6 = 0$

$\lambda = .8 \quad \lambda = 1.1$

saddle point

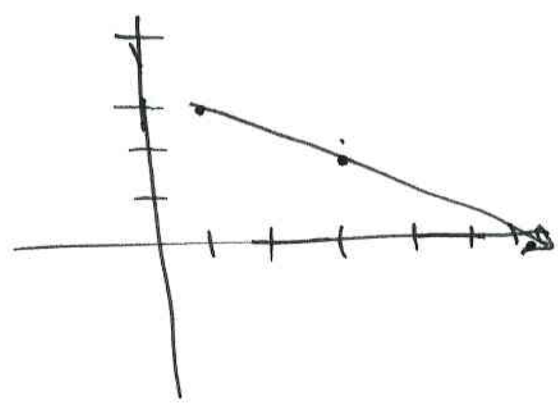
$\begin{bmatrix} 5 \\ 15 \end{bmatrix}, \begin{bmatrix} 11.5 \\ 19.5 \end{bmatrix}, \begin{bmatrix} 17.45 \\ 23.85 \end{bmatrix}, \begin{bmatrix} 23... \\ 28... \end{bmatrix}, \begin{bmatrix} 28.4... \\ 32.5... \end{bmatrix}$



7c. $\begin{bmatrix} 1.7-\lambda & .6 \\ -.4 & .7-\lambda \end{bmatrix}$ $(1.7-\lambda)(.7-\lambda) + .6 * .4 = 0$

$\lambda = 1.1 \quad \lambda = 1.3$

repeller



$\begin{bmatrix} 5 \\ 15 \end{bmatrix}, \begin{bmatrix} 17.5 \\ 8.5 \end{bmatrix}, \begin{bmatrix} 34.85 \\ -1.05 \end{bmatrix}, \begin{bmatrix} 58.615 \\ -14.675 \end{bmatrix}, \begin{bmatrix} 90.84 \\ -33.71 \end{bmatrix}, \ldots$