Math 268, Homework #5, Spring 2012 Name ____

Instructions: Write your work up neatly and attach to this page. Record your final answers (only) *directly on this page*. Use exact values unless specifically asked to round.

1. On a given quiz a student who receives an A has a 70% chance of getting an A on the next quiz, and a 20% chance of getting a B and 10% for any other grade. A student who gets a B on a quiz has a 50% chance of getting a B, a 10% chance of getting a A, and 40% chance of getting some other grade. A student who gets a grade less than B, has a 20% chance of getting a B, and a 5% chance of getting an A.
   a. Find the stochastic matrix that represents the scores in the class.
   b. If on one quiz, 80% of the students scored an A, 10% scored a B, and 10% scored below a B. What is the likely distribution of scores should the teacher expect on the next quiz?
   c. What is the long term distribution of scores in the class?

2. Find the steady state vector for the following stochastic matrices both algebraically and by calculator.
   a. $\begin{bmatrix} .1 & .7 \\ .9 & .3 \end{bmatrix}$
   b. $\begin{bmatrix} .6 & .2 & .04 \\ .25 & .5 & .01 \\ .15 & .3 & .95 \end{bmatrix}$

3. For each statement, indicate whether it's true or false. For the ones that are false, state the correct true statement.
   a. A stochastic matrix has a unique equilibrium vector.
   b. If $A\vec{x} = \lambda\vec{x}$ for some vector $\vec{x}$, then λ is an eigenvalue of A.
   c. A matrix is not invertible if and only if 0 is an eigenvalue of A.
   d. If $\vec{v_1}$ and $\vec{v_2}$ are linearly independent eigenvectors, then they correspond to distinct eigenvalues.
   e. A steady state vector for a stochastic matrix is actually an eigenvector.
   f. The eigenvalues of a matrix are on its main diagonal.
   g. An nxn matrix can have more than n eigenvalues.
   h. The determinant of A is the product of the diagonal entries of A.
   i. The elementary row operations of A do not change its determinant.
   j. If λ+5 is a factor of the characteristic polynomial, then 5 is an eigenvalue of A.

4. Find the characteristic polynomial and all eigenvalues and eigenvectors for each matrix.

   a. $\begin{bmatrix} 8 & 2 \\ 3 & 3 \end{bmatrix}$

   b. $\begin{bmatrix} -4 & 3 \\ 2 & 1 \end{bmatrix}$

   c. $\begin{bmatrix} 3 & 0 & 0 \\ 2 & 1 & 4 \\ 1 & 0 & 4 \end{bmatrix}$

   d. $\begin{bmatrix} 5 & 5 & 0 & 2 \\ 0 & 2 & -3 & 6 \\ 0 & 0 & 3 & -2 \\ 0 & 0 & 0 & 5 \end{bmatrix}$

   e. $\begin{bmatrix} 1 & -2 \\ 1 & 3 \end{bmatrix}$

   f. $\begin{bmatrix} 0 & 5 \\ -2 & 2 \end{bmatrix}$

   g. $\begin{bmatrix} -3 & -8 \\ 4 & 5 \end{bmatrix}$

5. In problem #4, e, f, g have complex eigenvalues. Use those complex eignenvalues to create a matrix P and C so that P is a similarity transformation and C is similar to the matrices in e, f and g.

6. In old-growth forests of Douglas fir, the spotted owl dines mainly on flying squirrels. Suppose the predator-prey matrix for these two populations is $A = \begin{bmatrix} .4 & .3 \\ -p & 1.2 \end{bmatrix}$. Show that if the predation parameter p is .325, both populations grow. Estimate the long-term growth rate and the eventual ratio of owls to flying squirrels. Repeat the problem with p=.104 and p=.855.

7. For each matrix below, classify the origin as an attractor, repeller or saddle point of the dynamical system $\overrightarrow{x_{k+1}} = A\overrightarrow{x_k}$. Find the direction of greatest attraction and repulsion. Plot the trajectories of the system starting from the $\overrightarrow{x_0} = \begin{bmatrix} 5 \\ 15 \end{bmatrix}$.

   a. $\begin{bmatrix} 1.7 & -.3 \\ -1.2 & .8 \end{bmatrix}$

   b. $\begin{bmatrix} .5 & .6 \\ -.3 & 1.4 \end{bmatrix}$

   c. $\begin{bmatrix} 1.7 & .6 \\ -.4 & .7 \end{bmatrix}$