Math 268, Homework #4, Spring 2012   Name ____

Instructions: Write your work up neatly and attach to this page. Record your final answers (only) *directly on this page*. Use exact values unless specifically asked to round.

1. For each of the sets of bases for R^3, determine which ones are linearly independent and which ones span R^3.

   a. $\begin{bmatrix}1\\0\\0\end{bmatrix},\begin{bmatrix}1\\1\\0\end{bmatrix},\begin{bmatrix}1\\1\\1\end{bmatrix}$

   b. $\begin{bmatrix}2\\-1\\1\end{bmatrix},\begin{bmatrix}2\\-3\\2\end{bmatrix},\begin{bmatrix}-8\\5\\4\end{bmatrix}$

   c. $\begin{bmatrix}1\\2\\-4\end{bmatrix},\begin{bmatrix}4\\3\\6\end{bmatrix}$

   d. $\begin{bmatrix}1\\-2\\3\end{bmatrix},\begin{bmatrix}0\\3\\-1\end{bmatrix},\begin{bmatrix}2\\-1\\5\end{bmatrix},\begin{bmatrix}0\\0\\-1\end{bmatrix}$

2. Find a basis for the space spanned by the given vectors.

   a. $\begin{bmatrix}1\\0\\-2\\3\end{bmatrix},\begin{bmatrix}0\\1\\2\\3\end{bmatrix},\begin{bmatrix}2\\-2\\-8\\0\end{bmatrix},\begin{bmatrix}2\\-1\\10\\3\end{bmatrix},\begin{bmatrix}3\\-1\\-6\\9\end{bmatrix}$

   b. $\begin{bmatrix}-3\\2\\6\\0\\-7\end{bmatrix},\begin{bmatrix}3\\0\\-9\\0\\6\end{bmatrix},\begin{bmatrix}0\\2\\-4\\0\\-1\end{bmatrix},\begin{bmatrix}6\\-2\\-14\\0\\13\end{bmatrix},\begin{bmatrix}-6\\3\\0\\-1\\0\end{bmatrix}$

3. For each statement below determine if it is true or false. If the statement is false, briefly explain why it is false and give the true statement. Assume $\mathscr{B}$ is a basis of the vector space V.

   a. A single vector by itself is linearly independent.

   b. If $H = span\{\vec{b}_1,\ldots,\vec{b}_n\}$, then $\{\vec{b}_1,\ldots,\vec{b}_n\}$ is a basis for H.

   c. The columns of an invertible *n*x*n* matrix for a basis for R^n.

d. The basis is a spanning set that is as large as possible.

e. In some cases, the linear independence relations among the columns of a matrix can be affected by certain elementary row operations of the matrix.

f. A linearly independent set in a subspace H is a basis for H.

g. If a finite set S of nonzero vectors spans a vector space V, then some subset of S is a basis for V.

h. The standard method for producing a spanning set for Nul A, described previously, sometimes fails to produce a basis for Nul A.

i. If B is an echelon form of a matrix A, then the pivot columns of B for a basis for Col A.

j. If $\vec{x}$ is in V and if $\mathscr{B}$ contains n vectors, then the $\mathscr{B}$-coordinate vector of $\vec{x}$ is in R^n.

k. If P_B is the change-of-coordinates matrix, then $\left[\vec{x}\right]_B = P_B\vec{x}$ for $\vec{x}$ in V.

l. The vector space $\mathbb{P}_3$ and R^3 are isomorphic.

m. If $\mathscr{B}$ is the standard basis for R^n, then the $\mathscr{B}$-coordinate vector of an $\vec{x}$ in R^n is $\vec{x}$ itself.

n. In some cases, a plane in R^3 can be isomorphic to R^2.

o. The row space of A is the same as the column space of A^T.

p. The sum of the dimensions of the row space and the null space of A equals the number of rows of A.

q. The dimensions of the null space of A is the number of columns of A that are not pivot columns.

r. If A and B are row equivalent, then their row spaces are the same.

s. Dim Row A + dim Nul A = n

t. The columns of the change-of-coordinate matrix $\underset{C \leftarrow B}{P}$ are B-coordinate vectors of the vectors in C.

u. The columns of $\underset{C \leftarrow B}{P}$ are linearly independent.

4. Find the vector $\vec{x}$ or $\left[\vec{x}\right]_B$ relative to the basis $\mathscr{B}$ (depending on which is missing) for the given basis $\mathscr{B}$. (In other words, if you are given $\vec{x}$ find $\left[\vec{x}\right]_B$, and if you are given $\left[\vec{x}\right]_B$, find $\vec{x}$.) Find the change of coordinate matrices for each case.

a. $\mathscr{B} = \left\{ \begin{bmatrix} 3 \\ -5 \end{bmatrix}, \begin{bmatrix} -4 \\ 6 \end{bmatrix} \right\}, \left[\vec{x}\right]_B = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$

b. $\mathscr{B} = \left\{ \begin{bmatrix} -2 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 4 \\ -1 \\ 3 \end{bmatrix} \right\}, \left[\vec{x}\right]_B = \begin{bmatrix} -3 \\ 2 \\ -1 \end{bmatrix}$

c. $\vec{b_1} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}, \vec{b_2} = \begin{bmatrix} 3 \\ -5 \end{bmatrix}, \vec{x} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

d. $\vec{b_1} = \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}, \vec{b_2} = \begin{bmatrix} 2 \\ 0 \\ 8 \end{bmatrix}, \vec{b_3} = \begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix}, \vec{x} = \begin{bmatrix} 0 \\ 0 \\ -2 \end{bmatrix}$

e. $\mathscr{B} = \left\{ \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \end{bmatrix} \right\}, \vec{x} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ (use an inverse matrix for this one)

f. $\mathscr{B} = \left\{1-t^2, t-t^2, 2-t+t^2\right\}$ for $\mathbb{P}_2$. Find $\vec{p}(t) = 1+3t-6t^2$ in this basis.

5. Find a basis for the subspace and state the dimension.

a. $\left\{ \begin{bmatrix} p-2q \\ 2p-5r \\ -2q+2r \\ -3p+6r \end{bmatrix} : p, q, r \in R \right\}$

b. $\left\{ \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ -6 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 3 \\ 5 \end{bmatrix}, \begin{bmatrix} -3 \\ 5 \\ 5 \end{bmatrix} \right\}$

c. $\begin{bmatrix} 1 & 2 & -4 & 3 & -2 & 6 & 0 \\ 0 & 0 & 0 & 1 & 0 & -3 & 7 \\ 0 & 0 & 0 & 0 & 1 & 4 & -2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$, for Col A and Nul A.

d. $\{1, 1-t, 2-4t+t^2, 6-18t+9t^2-t^3\}$

6. If $A=\begin{bmatrix}1&1&-2&-4&1&-2\\1&2&-3&0&-2&-3\\1&-1&0&0&1&6\\1&-2&2&1&-3&0\\1&-2&1&-4&2&-1\end{bmatrix}$ is row equivalent to $B=\begin{bmatrix}1&1&-2&-4&1&-2\\0&1&-1&0&-3&-1\\0&0&1&1&-13&-1\\0&0&0&0&1&-1\\0&0&0&0&0&1\end{bmatrix}$, find a basis for Col A, Row A and Nul A. Find rank A and dim Nul A without calculations.

7. Answer the following questions, and then explain why you know this to be the case. State a theorem of definition that applies.
   a. If a 7x5 matrix A has rank 2, find dim Nul A, dim Row A, and rank A^T.

   b. Suppose a 6x8 matrix A has 4 pivot columns. What is dim Nul A? Is Col A = R^4 ? Why or why not?

   c. If the null space of an 8x7 matrix is 5-dimensional, what is the dimension of the Col space of A?

   d. If A is a 5x4 matrix, what is the largest possible dimension of the row space of A?

   e. If A is a 7x5 matrix, what is the smallest possible dimension of Nul A?

   f. Suppose the solutions of a homogeneous system of 5 linear equations in 6 unknowns are all multiples of one nonzero solution. Will the system necessarily have a solution for every possible choice of constants on the right sides of the equations? Explain.

8. Find the change of coordinate matrices between the given bases.
   a. $\mathscr{B}=\left\{\begin{bmatrix}-2\vec{c_1}+4\vec{c_2}\\3\vec{c_1}+6\vec{c_2}\end{bmatrix}\right\}$, $\mathscr{C}$={c_1,c_2} be vector spaces for $\mathscr{V}$. Find $\underset{C\leftarrow B}{P}$ and $\left[\vec{x}\right]_B, \vec{x}=2\vec{b_1}+3\vec{b_2}$

   b. $\mathscr{B}=\left\{\begin{bmatrix}-1\\8\end{bmatrix},\begin{bmatrix}1\\-7\end{bmatrix}\right\}$, $\mathscr{C}=\left\{\begin{bmatrix}1\\2\end{bmatrix},\begin{bmatrix}1\\1\end{bmatrix}\right\}$. Find $\underset{C\leftarrow B}{P}$ and $\underset{B\leftarrow C}{P}$

   c. $\mathscr{F}=\left\{\begin{bmatrix}2\vec{d_1}-\vec{d_2}+\vec{d_3}\\3\vec{d_2}+\vec{d_3}\\-3\vec{d_1}+2\vec{d_3}\end{bmatrix}\right\}$, $\mathscr{D}$={d_1,d_2,d_3} be vector spaces for $\mathscr{V}$. Find $\underset{D\leftarrow F}{P}$ and $\left[\vec{x}\right]_D, \vec{x}=\vec{f_1}-2\vec{f_2}+2\vec{f_3}$

   d. In $\mathbf{P}_2$, $\mathscr{B}=\left\{1-2t+t^2, 3-5t+4t^2, 2t+3t^2\right\}$, $\mathscr{C}=\left\{1,t,t^2\right\}$, the standard basis. Find the B-coordinate vector for -1+2t.