Homework #3 Key Math 268 Spring 2012

1a. -14

b. $4\begin{bmatrix}5 & 2\\ 7 & 3\end{bmatrix} - 3\begin{bmatrix}6 & 2\\ 9 & 3\end{bmatrix} = 4$

c. $2\begin{bmatrix}0 & 0 & 5\\ 7 & 2 & -5\\ 3 & 1 & 8\end{bmatrix} = 2\cdot 5\begin{bmatrix}7 & 2\\ 3 & 1\end{bmatrix} = 10$

d. $-3\begin{bmatrix}3 & 2 & 4 & 0\\ 0 & -4 & 1 & 0\\ -5 & 6 & 7 & 1\\ 2 & 3 & 2 & 0\end{bmatrix} = -3(1)\begin{bmatrix}3 & 2 & 4\\ 0 & -4 & 1\\ 2 & 3 & 2\end{bmatrix} = -3\left[-4\begin{bmatrix}3 & 4\\ 2 & 2\end{bmatrix} - 1\begin{bmatrix}3 & 2\\ 2 & 3\end{bmatrix}\right] = 9$

e. $k\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} = k$

f. $-1\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} = -1$

2a. $\begin{bmatrix}1 & 5 & -3\\ 0 & -18 & 12\\ 0 & 3 & -1\end{bmatrix} = 1\begin{bmatrix}-18 & 12\\ 3 & -1\end{bmatrix} = -18$

b. $\begin{bmatrix}1 & -1 & -3 & 0\\ 0 & 1 & 5 & 4\\ 0 & 1 & 5 & 5\\ 0 & 2 & 7 & 3\end{bmatrix} = 1\begin{bmatrix}1 & 5 & 4\\ 1 & 5 & 5\\ 2 & 7 & 3\end{bmatrix} = 1\begin{bmatrix}1 & 5 & 4\\ 0 & 0 & 1\\ 0 & -3 & -5\end{bmatrix} = 1\cdot 1\begin{bmatrix}0 & 1\\ -3 & -5\end{bmatrix} = 3$

c. $\begin{bmatrix}1 & 3 & -1 & 0 & -2\\ 0 & 2 & -4 & -1 & -6\\ 0 & 0 & 0 & 3 & 5\\ 0 & -2 & -\bullet & 8 & -1\\ 0 & -4 & 8 & 2 & 13\end{bmatrix} = 1\begin{bmatrix}2 & -4 & -1 & -6\\ 0 & 0 & 3 & 5\\ -2 & -\bullet & 8 & -1\\ -4 & 8 & 2 & 13\end{bmatrix} = \begin{bmatrix}2 & 4 & -1 & -6\\ 0 & 0 & 3 & 5\\ 0 & -14 & 7 & -7\\ 0 & 0 & 0 & 1\end{bmatrix} =$

$-1\begin{bmatrix}2 & 4 & -1 & -6\\ 0 & -4 & 7 & -7\\ 0 & 0 & 3 & 5\\ 0 & 0 & 0 & 1\end{bmatrix} = -1(2)(-14)(3)(1) = 24$

2d. $-1\begin{bmatrix} 1 & 3 & 0 & -3 \\ -3 & -2 & 1 & -4 \\ -3 & 4 & -2 & 8 \\ 3 & -4 & 0 & 4 \end{bmatrix} = -1\begin{bmatrix} 1 & 3 & 0 & -3 \\ 0 & -6 & 1 & 0 \\ 0 & 0 & -2 & 12 \\ 0 & -13 & 0 & 13 \end{bmatrix} = -1(1)\begin{bmatrix} -6 & 1 & 0 \\ 0 & -2 & 12 \\ -13 & 0 & 13 \end{bmatrix} =$

$-1\left[-6\begin{bmatrix} -2 & 12 \\ 0 & 13 \end{bmatrix} - 1\begin{bmatrix} 0 & 12 \\ -13 & 13 \end{bmatrix}\right] = -1(-6)\left[-26\right] - 1(-1)\left[156\right] = -156 + 156 = 0$

3\. a. 21

b. -7

4\. #1: a-f is all invertible provided $k \neq 0$

#2: a-c invertible; 2d is not

5\. a. false; some do, some don't

b. false; $\det A \neq 0$

c. true

d. false; only if the matrix is triangular

e. false; one row could be a linear combination of others in the matrix

f. false; $\det A^T = \det A$

6.a. if A is invertible, then $AA^{-1} = I \Rightarrow \det(AA^{-1}) = \det I$

$\det I = 1 \Rightarrow \det A \cdot \det A^{-1} = 1 \Rightarrow \det A^{-1} = 1/\det A$

b. $\det(AB) = (\det A)(\det B) = (\det B)(\det A) = \det(BA)$

real #'s so commutativity applies

c. $\det(PAP^{-1}) = (\det P)(\det A)(\det P^{-1}) = (\det P)(\det P^{-1})(\det A) = \det(PP^{-1})\det A$

$= (\det I)\det A = 1 \det A = \det A$

d. $(\det A^4) = 0 \Rightarrow (\det A)^4 = 0 \Rightarrow \det A = 0$; thus A not invertible

e. i. $\det AB = (-1)(2) = -2$

ii. $\det(B^5) = 2^5 = 32$

e. iii. $\det(2A) = -2^n$

iv. $\det(A^T A) = \det A^T \cdot \det A = (-1)(-1) = 1$

v. $\det B^{-1}AB = \det A = -1$

7. a. yes. since sums are of this form, products & 0 (if $a=0$)

b. no since if we add 2 polynomials of this form, we get $(a+b) + 2t^2$ which is not of the required form.

& 0 not there.

c. yes.

d. yes. $p(0)=0 \Rightarrow$ no constants, but otherwise satisfies all conditions.

e. $W = b\begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix} + c\begin{bmatrix} 3 \\ 0 \\ 2 \end{bmatrix}$ $W = \text{Span}\left\{\begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ 2 \end{bmatrix}\right\}$

these are linearily independent, & subspace of $\mathbb{R}^3$

8 a. false. $\vec{0}$ must be 0 for all t

b. false. it can be, but it needn't be.

c. that is one condition (only), false

d. true

e. true

f. false. there are subspaces of $\mathbb{R}^3$ isomophic to $\mathbb{R}^2$

g. true

h. true

i. false. Nul A is in $\mathbb{R}^n$

j. true

k. true

l. true

m. true

9.a. $\begin{bmatrix} 1 & 2 & 4 & 0 \\ 0 & 1 & 3 & -2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & -2 & 4 \\ 0 & 1 & 3 & -2 \end{bmatrix}$

$x_1 = 2x_3 - 4x_4$
$x_2 = -3x_3 + 2x_4$
$x_3 = x_3$
$x_4 \;\; x_4$

$\rightarrow \vec{x} = x_3 \begin{bmatrix} 2 \\ -3 \\ 1 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} -4 \\ 2 \\ 0 \\ 1 \end{bmatrix}$

$\mathrm{Nul}\, A = \mathrm{Span} \left\{ \begin{bmatrix} 2 \\ -3 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -4 \\ 2 \\ 0 \\ 1 \end{bmatrix} \right\}$

b. $\begin{bmatrix} 1 & 3 & -4 & -3 & 1 \\ 0 & 1 & -3 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 5 & -6 & 1 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

$x_1 = -5x_3 + 6x_4 - 1x_5$
$x_2 = 3x_3$
$x_3 = x_3$
$x_4 = x_4$
$x_5 = x_5$

$\Rightarrow \vec{x} = x_3 \begin{bmatrix} -5 \\ 3 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 6 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} -1 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

$\mathrm{Nul}\, A = \mathrm{Span} \left\{ \begin{bmatrix} -5 \\ 3 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 6 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$