Homework #2 Key Math 268 Spring '12

1a. $\vec{b}$ is not a possible output vector of A.

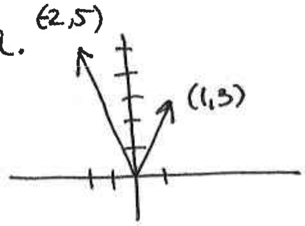
b. the row reduced augmented matrix is

$$\left[\begin{array}{ccc|c} 1 & 0 & 8 & 10 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$
$\uparrow$

there is one free variable so the solution is <u>not</u> unique

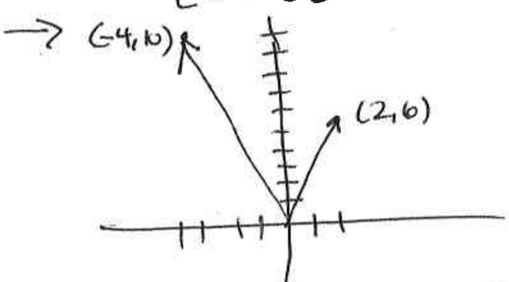
2. Since A $(m \times n)$ maps $\mathbb{R}^n \to \mathbb{R}^m$, the matrix must be 7×5.

3.a.



$\rightarrow$

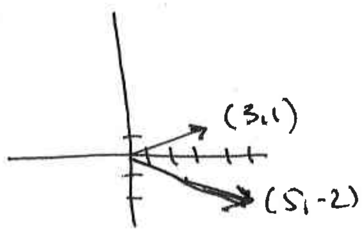
$\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 \\ 6 \end{bmatrix}$ $\quad$ $\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} -2 \\ 5 \end{bmatrix} = \begin{bmatrix} -4 \\ 10 \end{bmatrix}$



expansion/stretch by 2

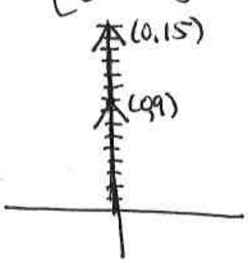
b. $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$ $\quad$ $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} -2 \\ 5 \end{bmatrix} = \begin{bmatrix} 5 \\ -2 \end{bmatrix}$

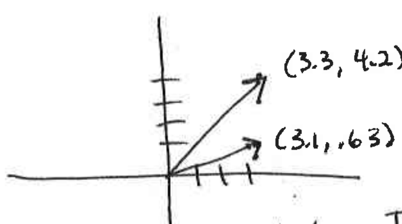
reflection around the $y = x$ line (switches coordinates)



c. $\begin{bmatrix} 0 & 0 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 0 \\ 9 \end{bmatrix}$ $\quad$ $\begin{bmatrix} 0 & 0 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} -2 \\ 5 \end{bmatrix} = \begin{bmatrix} 0 \\ 15 \end{bmatrix}$

projection onto y-axis + stretch by 3





d. $\cos \pi/3 = 1/2$ $\quad$ $\sin \pi/3 = \sqrt{3}/2$

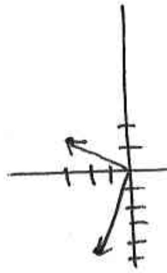
$\begin{bmatrix} 1/2 & \sqrt{3}/2 \\ -\sqrt{3}/2 & 1/2 \end{bmatrix}\begin{bmatrix} 1 \\ 3 \end{bmatrix} \approx \begin{bmatrix} 3.1 \\ .63 \end{bmatrix}$ $\quad$ $\begin{bmatrix} 1/2 & \sqrt{3}/2 \\ -\sqrt{3}/2 & 1/2 \end{bmatrix}\begin{bmatrix} -2 \\ 5 \end{bmatrix} \approx \begin{bmatrix} 3.3 \\ 4.2 \end{bmatrix}$

rotates $\pi/3$ clockwise

3d. $\cos 5\pi/4 = -1/\sqrt{2}$ $\sin(5\pi/4) = -1/\sqrt{2}$

$$\begin{bmatrix} -1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{bmatrix}\begin{bmatrix} 1 \\ 3 \end{bmatrix} = \begin{bmatrix} -4/\sqrt{2} \\ -2/\sqrt{2} \end{bmatrix} \approx \begin{bmatrix} -2.8 \\ -1.4 \end{bmatrix}$$

$$\begin{bmatrix} -1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{bmatrix}\begin{bmatrix} -2 \\ 5 \end{bmatrix} = \begin{bmatrix} -3/\sqrt{2} \\ -7/\sqrt{2} \end{bmatrix} \approx \begin{bmatrix} -2.1 \\ -4.9 \end{bmatrix}$$



rotation clockwise by $5\pi/4$

4a. false; the range of T is $\mathbb{R}^3$, A maps $\mathbb{R}^n \to \mathbb{R}^m$, so $\mathbb{R}^5 \to \mathbb{R}^3$

b. false; matrix transformations are linear, but not all linear transformations are matrixes (see derivatives)

c. true (see the definition of linear transformations)

5. The easiest way to show this is to show that $\vec{0}$ doesn't map to $\vec{0}$

$T(\vec{0}) = A(\vec{0}) + \vec{b} = \vec{b}$, but if $\vec{b} \neq 0$ then $T(\vec{0}) \neq \vec{0}$.

this is required of a linear transformation.

6. a. $\begin{bmatrix} 3 & -5 \\ 1 & 2 \\ 3 & 0 \\ 1 & 0 \end{bmatrix} = A$ check by multiplying $A\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ & $A\begin{bmatrix} 0 \\ 1 \end{bmatrix}$

b. $A = \begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix}$

c. $\cos(-3\pi/2) = \cos(\pi/2) = 0$
$\sin(-3\pi/2) = \sin(\pi/2) = 1$

$A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

d. one easy way to think of this is to separate the transformations & then multiply them so that $BC = A$

$B = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ $C = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

$\cos(-\pi/2) = 0$
$\sin(-\pi/2) = -1$

Counterclockwise rotation

$BC = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = A$ (same as 3b)

7a. true

b. true

c. 3x2 maps $\mathbb{R}^2 \to \mathbb{R}^3$ this can be 1-to-1 so its false

d. true (restatement of 7a)

8. 3a. one-to-one & onto (row-reduces to the identity)

3b. one-to-one & onto (row-reduces to the identity)

3c. neither one-to-one nor onto (there is only one pivot)

3d. one-to-one and onto (row-reduces to the identity)

3e. one-to-one and onto (row-reduces to the identity)

6a. one-to-one (pivots in both columns), <u>not</u> onto (not enough pivots in rows)

6b. one-to-one and onto (row-reduces to the identity)

6c. one-to-one and onto (row-reduces to the identity)

6d. one-to-one and onto (row-reduces to the identity)

9. a.

$$\begin{aligned} 50 &= 11 I_1 - 5 I_2 \\ -40 &= -5 I_1 + 10 I_2 - I_3 \\ 30 &= -I_2 + 9 I_3 - 2 I_4 \\ -30 &= -2 I_3 + 10 I_4 \end{aligned}$$

$I_1 \approx 3.7$
$I_2 \approx -1.9$
$I_3 \approx 2.6$
$I_4 \approx -2.5$

b.

$$\begin{aligned} 40 &= 12 I_1 - 7 I_2 - 4 I_4 \\ 30 &= -7 I_1 + 15 I_2 - 6 I_3 \\ 20 &= -6 I_2 + 14 I_3 - 5 I_4 \\ -10 &= -4 I_1 - 5 I_3 + 13 I_4 \end{aligned}$$

$I_1 \approx 11.4$
$I_2 \approx 10.6$
$I_3 \approx 8.0$
$I_4 \approx 5.8$

9c. $9I_1 - I_2 \qquad - I_4 - 4I_5 = 50$

$-I_1 + 7I_2 - 2I_3 \qquad - 3I_5 = -30$

$-2I_2 + 10I_3 - 3I_4 - 3I_5 = 20$

$-I_1 \qquad - 3I_3 + 7I_4 - 2I_5 = -40$

$-4I_1 - 3I_2 - 3I_3 - 2I_4 + 12I_5 = 0$

$I_1 \approx 4$
$I_2 \approx -4.4$
$I_3 \approx -.9$
$I_4 \approx -5.8$
$I_5 \approx -1$

10. $\begin{bmatrix} \text{West} \\ \text{North} \\ \text{East} \end{bmatrix} = \begin{bmatrix} 295 \\ 55 \\ 400 \end{bmatrix}$ Monday

$A \begin{bmatrix} 295 \\ 55 \\ 400 \end{bmatrix} = \begin{bmatrix} 343 \\ 131 \\ 276 \end{bmatrix}$ Tuesday

$A \begin{bmatrix} 343 \\ 131 \\ 276 \end{bmatrix} = \begin{bmatrix} 374 \\ 174 \\ 202 \end{bmatrix}$ Wednesday

Monday (next) is 7 days after the initial info so we need A^7 (matrix A times itself 7 times) times the initial values

$A^7 \begin{bmatrix} 295 \\ 55 \\ 400 \end{bmatrix} = \begin{bmatrix} 431 \\ 217 \\ 102 \end{bmatrix}$

you may get slightly different numbers if you multiply & round after each step, but you won't be off by much. (a car or two)

11a. i. $A + B = \begin{bmatrix} 12 & 4 \\ 0 & 4 \end{bmatrix}$

ii. $2B - 3C = \begin{bmatrix} 12 & 12 \\ 14 & -12 \end{bmatrix}$

iii. $D + E = \begin{bmatrix} 1 & 0 & 9 \\ -1 & -3 & 0 \\ 2 & -2 & -6 \end{bmatrix}$

11b. i. $4A = \begin{bmatrix} 12 & 4 \\ -4 & 16 \end{bmatrix}$

ii. $2F = \begin{bmatrix} 2 & 6 & -4 & 0 \\ 4 & 8 & -2 & 10 \end{bmatrix}$

iii. $21G = \begin{bmatrix} 126 & -147 \\ 231 & -105 \\ 42 & 63 \end{bmatrix}$

iv. $-5H = \begin{bmatrix} -5 & 0 & 10 \end{bmatrix}$

c. i. $AB = \begin{bmatrix} 28 & 9 \\ -5 & -3 \end{bmatrix}$

ii. $\begin{bmatrix} 24 & 21 \\ 3 & 1 \end{bmatrix} = BA$

iii. $DE = \begin{bmatrix} -1 & -7 & -23 \\ 1 & 2 & -10 \\ -5 & 9 & 8 \end{bmatrix}$

iv. $BF = \begin{bmatrix} 15 & 39 & -21 & 15 \\ 1 & 3 & -2 & 0 \end{bmatrix}$

v. $GC = \begin{bmatrix} 40 & -40 \\ 42 & -42 \\ -8 & 8 \end{bmatrix}$

vi. $DG = \begin{bmatrix} 47 & -10 \\ -1 & 9 \\ -24 & 2 \end{bmatrix}$

vii. dimension mismatch

viii. $\begin{bmatrix} 33 \\ 4 \end{bmatrix}$

d. i. $A^T = \begin{bmatrix} 3 & -1 \\ 1 & 4 \end{bmatrix}$

ii. $D^T = \begin{bmatrix} 1 & -2 & 3 \\ 3 & 1 & -4 \\ 4 & 0 & 1 \end{bmatrix}$

iii. $G^T = \begin{bmatrix} 6 & 11 & 2 \\ -7 & -5 & 3 \end{bmatrix}$

iv. $J^T = \begin{bmatrix} 4 & -1 \end{bmatrix}$

e. i. A^{-1} $\begin{bmatrix} 4/13 & -1/13 \\ 1/13 & 3/13 \end{bmatrix}$

ii. C^{-1} = not invertible

11.e. iii. $D^{-1} = \begin{bmatrix} 1/27 & -19/27 & -4/27 \\ 2/27 & -11/27 & -8/27 \\ 5/27 & 13/27 & 7/27 \end{bmatrix}$ iv. F is not square no inverse

12.a. $\begin{bmatrix} 2 & 3 \\ 4 & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 12 \\ 10 \end{bmatrix}$ $A^{-1}\begin{bmatrix} 12 \\ 10 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

b. $\begin{bmatrix} -1 & 5 \\ 3 & -4 \end{bmatrix}\begin{bmatrix} \cancel{x_1} \\ y \end{bmatrix} = \begin{bmatrix} 17 \\ 12 \end{bmatrix}$ $A^{-1}\begin{bmatrix} 17 \\ 12 \end{bmatrix} = \begin{bmatrix} 128/11 \\ 63/11 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

c. $\begin{bmatrix} 5 & -1 & 2 \\ 3 & 2 & -4 \\ -4 & -3 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ 16 \\ 7 \end{bmatrix}$ $A^{-1}\begin{bmatrix} 10 \\ 16 \\ 7 \end{bmatrix} = \begin{bmatrix} 36/13 \\ -8 \\ -77/13 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

d. $\begin{bmatrix} 1 & 1 & 1 \\ -1 & 2 & -3 \\ 3 & -5 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 14 \\ -18 \end{bmatrix}$ $A^{-1}\begin{bmatrix} 9 \\ 14 \\ -18 \end{bmatrix} = \begin{bmatrix} 123/31 \\ 205/31 \\ -49/31 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

13a. true

b. false both can be reduced to the identity I_n but the operations have to be performed in reverse: add where before you subtracted, or multiply by k where before by 1/k, etc.

c. false $(AB)^{-1} = B^{-1}A^{-1}$

d. true

e. true

f. true

g. true

h. true

i. false. only if A is one-to-one

14. a. not invertible (rows are multiples)

b. not invertible (row of zeros)

c. invertible (pivot in every column (& row))

d. invertible