Homework #1 Math 268

1\. a. $\begin{bmatrix} 3 & 6 & -3 \\ 5 & 7 & 10 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 9 \\ 0 & 1 & -5 \end{bmatrix}$ $\vec{x} = \begin{bmatrix} 9 \\ -5 \end{bmatrix}$

consistent, independent

b. $\begin{bmatrix} 2 & 0 & -6 & -8 \\ 0 & 1 & 2 & 3 \\ 3 & 6 & -2 & -4 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$ $\vec{x} = \begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix}$

consistent, independent

c. $\begin{bmatrix} 2 & 0 & 0 & -4 & -10 \\ 0 & 3 & 3 & 6 & 0 \\ 0 & 0 & 1 & 4 & -1 \\ -3 & 2 & 3 & 1 & 5 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 & -3 \\ 0 & 1 & 0 & 0 & 5 \\ 0 & 0 & 1 & 0 & -5 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$

$\vec{x} = \begin{bmatrix} -3 \\ 5 \\ -5 \\ 1 \end{bmatrix}$ consistent, independent

2\. a. $\begin{bmatrix} 1 & h & -5 \\ 2 & -8 & 6 \end{bmatrix}$ $-2R_1 + R_2 \rightarrow R_2$ $\begin{bmatrix} 1 & h & -5 \\ 0 & -2h-8 & 16 \end{bmatrix}$ $\frac{1}{-2h-8} R_2 \rightarrow R_2$

$\begin{bmatrix} 1 & h & -5 \\ 0 & 1 & \frac{16}{-2h-8} \end{bmatrix}$ this system will be consistent as long as $-2h-8 \neq 0$

$\frac{2h}{2} \neq \frac{-8}{2} \Rightarrow h \neq -4$

b. $\begin{bmatrix} -4 & 12 & h \\ 2 & -6 & -3 \end{bmatrix}$ $2R_2 + R_1 \rightarrow R_2$ $\begin{bmatrix} -4 & 12 & h \\ 0 & 0 & -6+h \end{bmatrix}$

to be consistent $-6+h = 0 \Rightarrow h = 6$

c. $\begin{bmatrix} 1 & -4 & 7 & g \\ 0 & 3 & -5 & h \\ -2 & 5 & -9 & k \end{bmatrix}$ $2R_1 + R_3 \rightarrow R_3$ $\begin{bmatrix} 1 & -4 & 7 & g \\ 0 & 3 & -5 & h \\ 0 & -3 & 5 & 2g+k \end{bmatrix}$ $R_2 + R_3 \rightarrow R_3$

$\begin{bmatrix} 1 & -4 & 7 & g \\ 0 & 3 & -5 & h \\ 0 & 0 & 0 & 2g+h+k \end{bmatrix}$ to be consistent $2g + h + k = 0$

3. a. True (as long as you haven't multiplied by 0)
   b. true
   c. true
   d. false; two equivalent systems have identical solution sets

4. a. $\begin{bmatrix} 1 & 2 & 4 & 5 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 1 & 2 \end{bmatrix}$ (row) echelon form; this form is not unique as it will depend on row operations only the last row (last pivot row) will agree ∴ answers will vary somewhat.

$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 2 \end{bmatrix}$ reduced (row) echelon form is unique

b. this matrix is already in echelon form
if this was an augmented matrix representing a system it would be inconsistent, but in other situations it may still be worth reducing further.

$\begin{bmatrix} 1 & 0 & -9 & 0 & 0 \\ 0 & 1 & 3 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$ reduced (row) echelon form

5. $\begin{bmatrix} 1 & * & * & * & * \\ 0 & 1 & * & * & * \\ 0 & 0 & 1 & * & * \\ 0 & 0 & 0 & 1 & * \end{bmatrix}$ consistent, independent

$\begin{bmatrix} 1 & * & * & * & * \\ 0 & 0 & 1 & * & * \\ 0 & 0 & 0 & 1 & * \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$ inconsistent

$\begin{bmatrix} 1 & * & * & * & * \\ 0 & 1 & * & * & * \\ 0 & 0 & 0 & 1 & * \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ consistent, dependent

$\begin{bmatrix} 0 & 1 & * & * & * \\ 0 & 0 & 1 & * & * \\ 0 & 0 & 0 & 1 & * \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$ inconsistent

$\begin{bmatrix} 0 & 1 & * & * & * \\ 0 & 0 & 1 & * & * \\ 0 & 0 & 0 & 1 & * \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ consistent, dependent

answers will vary

6\. a. False. we can apply it to vectors or collections of them to determine ③ if they are linearly independent, and other things.

b. True, for dependent systems.

c. False. If one row was $[0 \ 0 \ 0 \ 0 \ 5]$ then it would imply inconsistency. This just implies that $x_4 = 0$

d. False; in an augmented matrix, the last column should not have a pivot or its inconsistent.

e. False. The pivot positions are unique.

f. True.

7\. $\begin{cases} a_0 + a_1 + a_2 + a_3 = 7 \\ a_0 + 2a_1 + 4a_2 + 8a_3 = 17 \\ a_0 + 3a_1 + 9a_2 + 27a_3 = 31 \\ a_0 + 4a_1 + 16a_2 + 64a_3 = 65 \end{cases}$ $\begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} -15 \\ 100/3 \\ -14 \\ 8/3 \end{bmatrix}$

$$p(t) = -15 + \frac{100}{3}t - 14t^2 + \frac{8}{3}t^3$$

8\. a. $x_1 \begin{bmatrix} 3 \\ 5 \end{bmatrix} + x_2 \begin{bmatrix} 6 \\ 7 \end{bmatrix} = \begin{bmatrix} -3 \\ 10 \end{bmatrix}$ $\begin{bmatrix} 3 & 6 \\ 5 & 7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -3 \\ 10 \end{bmatrix}$

b. $x_1 \begin{bmatrix} 2 \\ 0 \\ 3 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 1 \\ 6 \end{bmatrix} + x_3 \begin{bmatrix} -6 \\ 2 \\ -2 \end{bmatrix} = \begin{bmatrix} -8 \\ 3 \\ -4 \end{bmatrix}$ $\begin{bmatrix} 2 & 1 & -6 \\ 0 & 0 & 2 \\ 3 & 6 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -8 \\ 3 \\ -4 \end{bmatrix}$

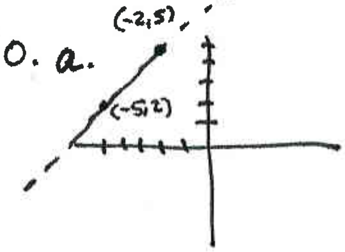
c. $x_1 \begin{bmatrix} 2 \\ 0 \\ 0 \\ -3 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 3 \\ 0 \\ 2 \end{bmatrix} + x_3 \begin{bmatrix} 0 \\ 3 \\ 1 \\ 3 \end{bmatrix} + x_4 \begin{bmatrix} -4 \\ 0 \\ 4 \\ 1 \end{bmatrix} = \begin{bmatrix} -10 \\ 0 \\ -1 \\ 5 \end{bmatrix}$ $\begin{bmatrix} 2 & 0 & 0 & -4 \\ 0 & 3 & 3 & 0 \\ 0 & 0 & 1 & 4 \\ -3 & 2 & 3 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -10 \\ 0 \\ -1 \\ 5 \end{bmatrix}$

9\. answers will vary

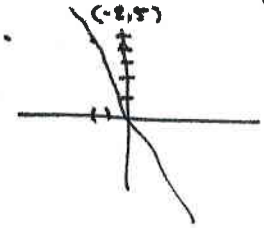
$\vec{u} = \begin{bmatrix} 1 \\ 3 \\ -2 \\ 0 \end{bmatrix}, \vec{v} = \begin{bmatrix} -4 \\ 9 \\ -1 \end{bmatrix}$

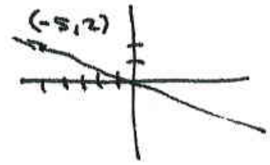
$2\vec{u} = \begin{bmatrix} 2 \\ 6 \\ -4 \\ 0 \end{bmatrix}, \quad -\vec{v} = \begin{bmatrix} 4 \\ 0 \\ -1 \\ 1 \end{bmatrix} \quad 2\vec{u}+\vec{v} = \begin{bmatrix} -2 \\ 6 \\ -3 \\ -1 \end{bmatrix}$

$\vec{u}+\vec{v} = \begin{bmatrix} -3 \\ 3 \\ -1 \\ -1 \end{bmatrix} \quad 2\vec{u}-\vec{v} = \begin{bmatrix} 6 \\ 6 \\ -5 \\ 1 \end{bmatrix}$

10. a. 

the line through the origin containing both points does not line through the origin. Each one individually does.





b. true

c. false. The list must be ordered.

d. true

e. false. $A\vec{x} = \vec{b}$ is a matrix equation.

f. false. The last row could be all zeros.

g. true.

11. a. reduced (row) echelon form $\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $\begin{matrix} \leftarrow \\ \leftarrow \\ \leftarrow \\ \end{matrix}$ 3 pivots only does not span $\mathbb{R}^4$

b. reduced (row) echelon form $\begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$ $\begin{matrix} \leftarrow \\ \leftarrow \\ \leftarrow \\ \leftarrow \end{matrix}$ 4 pivots <u>does</u> span $\mathbb{R}^4$

12. a. reduced (row) echelon form $\begin{bmatrix} 1 & 0 & 22/23 & 0 \\ 0 & 1 & -28/23 & 0 \end{bmatrix}$

$$\begin{array}{cccc} & & \underset{\text{free}}{\uparrow} & \end{array}$$

$$x_1 + \frac{22}{23}x_3 = 0 \rightarrow x_1 = -\frac{22}{23}x_3$$

$$x_2 + \frac{-28}{23}x_3 = 0 \rightarrow x_2 = \frac{28}{23}x_3 \rightarrow \vec{x} = \begin{bmatrix} -22/23 \\ 28/23 \\ 1 \end{bmatrix} x_3$$

$$x_3 = x_3$$

b. $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$ only the trivial solution exists

12c. $\begin{bmatrix} 1 & 0 & 0 & 0 & 97/54 & 0 \\ 0 & 1 & 0 & 0 & -101/27 & 0 \\ 0 & 0 & 1 & 0 & 92/27 & 0 \\ 0 & 0 & 0 & 1 & 35/54 & 0 \end{bmatrix}$

$\uparrow$ free

$x_1 + \frac{97}{54}x_5 = 0 \rightarrow x_1 = -\frac{97}{54}x_5$

$x_2 - \frac{101}{27}x_5 = 0 \rightarrow x_2 = \frac{101}{27}x_5$

$x_3 + \frac{92}{27}x_5 = 0 \rightarrow x_3 = -\frac{92}{27}x_5$

$x_4 + \frac{35}{54}x_5 = 0 \quad x_4 = -\frac{35}{54}x_5$

$x_5 = x_5$

$$\vec{x} = \begin{bmatrix} -97/54 \\ 101/27 \\ -92/27 \\ -35/54 \\ 1 \end{bmatrix} x_5$$

13. a. true

b. false. it has a non-trivail solution iff there is a free variable.

c. false. it is a line through p parallel to $\vec{v}$.

d. true

14. if $\begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$ is a solution, then $\begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} t$ is a solution.

$\Rightarrow$ $x_1 = 2x_3$, $x_2 = -x_3$, $x_3 = x_3$ $\rightarrow$ $x_1 - 2x_3 = 0$, $x_2 + x_3 = 0$

$\downarrow$

$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

15. a. $\begin{bmatrix} 1 & * \\ 0 & 1 \end{bmatrix}$

b. $\begin{bmatrix} 1 & * & * \\ 0 & 1 & * \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

if $\vec{a}_1, \vec{a}_2$ are linearly independent & $\vec{a}_3$ is not in the span, then all 3 are linearly independent

c. it must have 4 pivots since pivots in every column is necessary for linear independence.