Identifying Polynomials

Polynomials are algebraic expressions that contain whole number powers of a variable. The formal mathematical expression for this is $a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$. This looks intimidating, but the *a* terms are standing in for real numbers. So things that conform to this pattern are things like these

1 (here *n* is 0 so there is only the constant term)
2x+1 (here *n* is 1 so we have a linear expression)
$3x^2 + 2x - 1$ (here *n* is 2 so we have what we call a quadratic term, or a degree-2 polynomial)

When there is *only one variable* in polynomial, then the *n*, the highest power in the polynomial, is the **degree** of the polynomial.

Determining the degree of a polynomial when there are more than one variable is a little more difficult.

Example 1A. $4x^4y^6$
The degree of a polynomial that has more than one variable depends on the sum of the powers of all the variables in each term. The degree of this polynomial is 10 since 4+6=10.

Example 1B. $2x^2y^5 - 3xy^7 + x^3y^2$
First, look at each term separately and then choose the highest number. Remember that for variables without a power, the understood exponent is 1 and it has to be included in the count.

The first term has exponent 2+5=7.
The second term has exponents 1+7=8.
The third term has exponents 3+2=5.

The highest degree term is 8, and so the degree of the whole polynomial is 8.

Constants have degree zero, since we can write 1 as x^0. When they are multiplied by other variables we can ignore it, but if it's the only term, we say that **a constant is a polynomial of degree 0**. Unless the constant is zero: then we say that zero has *no degree*.

Sometimes it can help to compare polynomials with things that are not polynomials to get a better feel for them. Below are some examples of both.

Polynomials	Non-Polynomials
3	$\frac{1}{x}$
3x-2	$\frac{1}{3x-2}$
$6x^4 + 2x$	x^{-4}

$xy^2 - 3xy$	$\frac{y}{x}$
$x^5 + 2x^4 - 5x^3 + 6x^2$	$\lvert x^2 - 1\rvert$
$x^2 + y^2 + 4$	$\sqrt{x-2}$
$1 - 13x + x^{22}$	$x^{0.1}$

The degrees of the polynomials listed, in order, are 0, 1, 4, 3, 5, 2, 22.

Some polynomials appear so frequently in problems that they have special names.

Monomial: is a polynomial with only one term. Sometimes polynomials are defined in terms of sums of monomials.

Binomial: a polynomial with only two terms.

Trinomial: a polynomial with exactly three terms.

Anything with more than three terms is just a polynomial: they have no special name. But monomials, binomials and trinomials, even though they have special names, are still polynomials. Polynomials define the general class.

In the table above, only the first term in a monomial, as is the polynomial in Example 1A. The table, the second, third and fourth examples are binomials. The last two examples in the table, together with Example 1B are trinomials.

Practice Problems.
For each problem, determine the degree of the polynomial. Then determine if the polynomial can be referred to as a monomial, binomial or trinomial. If the problem is not a polynomial, so state.

1. $x^2 - 7x + 3$
2. 0
3. $6x^4 + 3x$
4. $|x + 5|$
5. $\frac{1}{3}x^2 + \frac{9}{7}x - \frac{1}{4}$
6. 3x-1
7. $\sqrt{x^4 - 1}$
8. $xy - 5$
9. $x^3z^2 - 17xz^5$
10. $(4x + 2)^{-2}$
11. $8x^2y^3z^3 - 9x^4y^2z^2 + z^7$
12. $(3x - 1)(4x + 2)$
13. $\frac{1}{2}xyz + 2yz - 3$
14. $14x^4 - 13x$