Percent Increase/Decrease Problems

Percent increase and percent decrease problems are very similar to each other. The only difference is that percent increase problems use a + (plus) sign where the percent decrease problems use a – (minus) sign. The general format of these problems looks like this:

OriginalValue + Increase = NewValue

Or

OriginalValue – Decrease = NewValue

In each case the Increase or Decrease is calculated by the Percent (in decimal form) times the Original Value. So we get the equation:

OriginalValue ± Percent*OriginalValue = NewValue

Where the ± sign indicates we use the sign appropriate for the problem we are doing.

This equation is common sense when the original value is provided up front and what we are seeking to calculate is the new value.

Example 1. Suppose that you are looking at a new dress marked at $80, but the racked is marked as 30% off list price. How much money will the dress cost at the register?

The original value of the dress is $80. And the percent decrease is 30%. Plug these values into the equation above.

80 – 0.30*80 = NewValue

We can plug this into our calculator to find 80 – 0.30*80 = 80 – 24 = 56. So our new value is $56. (Decrease)

Example 2. At the hardware store, you pick up a hammer and other miscellaneous items totaling $150. Tax in the area is 6%. How much will you be charged for your items?

Here, we are given the original value again, and so we use the same equation as before. Tax is a percent increase, so we use the plus sign.

150 + 0.06*150 = NewValue

Again, we go to the calculator to obtain the new value: 150 + 0.06*150 = 150 + 9 = 159. The charge to your credit card will be $159. (Increase)

These problems are less intuitive when we are given the New Value and not the original value. However, we will still use the same formula. Since the Original Value is unknown, we will replace it with "x" so that we can solve for it with algebra.

Example 3A. You are purchasing textbooks at a bookstore and the amount charged to your credit card is $187.25. If the tax in the county where you purchased the books is 7%, what was the pre-tax price of the books?

Since we are given the value after tax has been applied, this is the "new value". Replace the original value in the equation with x since that is what we are looking for.

OriginalValue + Percent*OriginalValue = New Value

x + 0.07x = 187.25

It's tempting to treat this as a percent decrease problem, but we will show why this doesn't work below. The 7% is based on the original price, not 7% of the new one.

In the equation x + 0.07x = 187.25, we need to add the 2 terms with x in them together in order to solve the equation. The understood coefficient of x is 1, so we have

(1+0.07)x = 187.25, or
1.07x = 187.25

Divide by 1.07 to find x.

$$x = \frac{187.25}{1.07} = 175$$

So the price of the books before tax was $175. We can check this in the same equation by putting $175 in for the original price: 175 + 0.07*175 = 187.25, which is what we were given.

Example 3B. For the problem above, sometimes students will try to treat this as a percent decrease problem and assume the following: New Value – Percent*NewValue = OriginalValue. But this equation will not work when we use the correct forward equation on the result. Since it doesn't give us the right answer, *the method I illustrate here cannot be used to solve problems of this type*.

187.25 – 0.07*187.25 = 174.1425

But: 174.1425 + 0.07*174.1425 = 186.332475 ≠ 187.25.

Again, let me stress, this method does not work; it gives the wrong answer and so cannot be used.

There is one and only one equation that can be used to solve these problems. Percent changes are always calculated based on the value that occurs **earlier in time**, never on the one that occurs later in time.

Example 4. During a period of hard economic times, Bob, a salesman, has his salary by 10% to help keep his company afloat. If his salary after the cut is $40,500 per year, what was his salary last year?

This is a percent decrease problem and so we use the percent decrease formula. The unknown value is the older salary and so "original value" is replaced by x.

OriginalValue – Percent*OriginalValue = NewValue

x – 0.10x = 40,500.

As in Example 3A, we combine the 0.10 with the understood 1 in front of the first x.

(1-0.10)x = 40,500
0.90x = 40,500

Divide by the 0.90 to find x.

$$x = \frac{40,500}{0.90} = 45,000$$

His old salary was $45,000.

Example 5. It's also possible to have a problem where we are asked to solve for the percent. As with the previous examples, we'll use the same formula.

Suppose that the population of a certain town was 105,000 in 2010 and 102,010 in 2011. Find the percent decrease between 2010 and 2011.

Even if we hadn't been told this was a decrease problem, we know it is because the old value (from 2010) is larger than the new value (from 2011). Since it's the percent we don't know, that's where x goes.

OriginalValue – Percent*OriginalValue = NewValue

105,000 – x*105,000 = 102,010

We can solve the equation for x now:

105,000 – 105,000x = 102,010
-105,000 -105,000 Subtract 105,000 from both sides

-105,000x = -2990

Divide by -105,000

$$x = -\frac{2990}{-105,000} = 0.028476\ldots$$

As a percent (multiply by 100), this is 2.8476…%, or taking one digit after the decimal in this form, approximately 2.8%.

Practice Problems.

Round all values in dollars to the nearest penny. Round all percents to one digit after the decimal place.

1. The list prices on three books totals $205.00. The tax in the local city is 5.5%. Find the amount charged to your credit card.
2. A sale rack lists a leather jacket for $300. The rack says the item is 45% off at the register. How much will the customer pay?
3. Suppose you purchase a car listed at $15,000, but the dealership is having a sale for 10% off. If tax in that county is 6%, what is the final price of the car when you drive it off the lot? [Hint: first calculate the discount, and then the tax on that value.]
4. The town of Fishkill had 43,231 people living in it in 2009. In 2011, that figure had gone up to 47,833. Find the percentage change in the population. Is this an increase or a decrease in the population? If the trend continues, what is the expected population in 2013?
5. If the tax in the city is 6.5% and your receipt says you were charged $201.24 for your purchase, what was the price of your items before tax?
6. If the standard retail mark-up is 35% and you purchased the item for $35.99, what was the likely wholesale price of the item?
7. Tammy needs a dress for the prom, but she has only $500 to spend on it. If she finds a store with a 70% off sale, how much can the list-price of the dress be if she stays under her $500 limit?
8. If a sales manager is hired for $35,000 a year and is guaranteed a 4% raise every year for the next 5 years, how much will his salary be at the end of those 5 years?