Derivative Tables

This handout contains tables of derivatives for some of the harder functions that you can check your work against. Be aware that you won't have this table available to you on any exams or quizzes, so be sure you can match the results of the tables.

Practice: For each of the derivative tables included in this handout, do the following:

i) Check the first five derivatives by hand.

ii) Determine a general formula (if possible) for the nth derivative. The program that generated these functions did a brute force approach. You may be able to do some algebraic simplification (particularly on the sec(x) and tan(x) functions using trig identities) to make the expressions more amenable to writing a formula.

iii) Use the shorter tables to generate a formula for the values of the derivatives at the given points. You may find it helpful to simplify here as well. Use your calculator to write the $\sqrt{x}$ values as fractions, and sec(x) may have a multiplier of $\sqrt{2}$ that you can divide out. Check your formulas against the next derivative in the longer table.

iv) Write the fifth-degree Taylor polynomial for each function centered at the given point.

1.

f(x) = $\sqrt{x}$

x–value x

order of derivative

$f(x) =$	$\sqrt{x}$
$f'(x) =$	$\frac{1}{2\sqrt{x}}$
$f''(x) =$	$-\frac{1}{4x^{3/2}}$
$f'''(x) =$	$\frac{3}{8x^{5/2}}$
$f^{(4)}(x) =$	$-\frac{15}{16x^{7/2}}$
$f^{(5)}(x) =$	$\frac{105}{32x^{9/2}}$
$f^{(6)}(x) =$	$-\frac{945}{64x^{11/2}}$
$f^{(7)}(x) =$	$\frac{10395}{128x^{13/2}}$
$f^{(8)}(x) =$	$-\frac{135\,135}{256x^{15/2}}$
$f^{(9)}(x) =$	$\frac{2\,027\,025}{512x^{17/2}}$
$f^{(10)}(x) =$	$-\frac{34\,459\,425}{1024x^{19/2}}$

f(x) = $\sqrt{x}$

x–value 1

order of derivative

$f(1.) =$	1.
$f'(1.) =$	0.5
$f''(1.) =$	−0.25
$f'''(1.) =$	0.375
$f^{(4)}(1.) =$	−0.9375
$f^{(5)}(1.) =$	3.28125

2.

f(x) = ln(x)

x–value x

order of derivative

f(x) =	$\log(x)$
f′(x) =	$\frac{1}{x}$
f″(x) =	$-\frac{1}{x^2}$
f‴(x) =	$\frac{2}{x^3}$
$f^{(4)}(x)$ =	$-\frac{6}{x^4}$
$f^{(5)}(x)$ =	$\frac{24}{x^5}$
$f^{(6)}(x)$ =	$-\frac{120}{x^6}$
$f^{(7)}(x)$ =	$\frac{720}{x^7}$
$f^{(8)}(x)$ =	$-\frac{5040}{x^8}$
$f^{(9)}(x)$ =	$\frac{40320}{x^9}$
$f^{(10)}(x)$ =	$-\frac{362880}{x^{10}}$

f(x) = ln(x)

x–value 1

order of derivative

f(1.) =	0.
f′(1.) =	1.
f″(1.) =	−1.
f‴(1.) =	2.
$f^{(4)}(1.)$ =	−6.
$f^{(5)}(1.)$ =	24.

3.

f(x) = e^{x^2}

x–value x

order of derivative

f(x) =	e^{x^2}
f′(x) =	$2 e^{x^2} x$
f″(x) =	$4 e^{x^2} x^2 + 2 e^{x^2}$
f‴(x) =	$12 e^{x^2} x + 8 e^{x^2} x^3$
$f^{(4)}(x)$ =	$48 e^{x^2} x^2 + 12 e^{x^2} + 16 e^{x^2} x^4$
$f^{(5)}(x)$ =	$120 e^{x^2} x + 32 e^{x^2} x^5 + 160 e^{x^2} x^3$
$f^{(6)}(x)$ =	$720 e^{x^2} x^2 + 120 e^{x^2} + 64 e^{x^2} x^6 + 480 e^{x^2} x^4$
$f^{(7)}(x)$ =	$1680 e^{x^2} x + 128 e^{x^2} x^7 + 1344 e^{x^2} x^5 + 3360 e^{x^2} x^3$
$f^{(8)}(x)$ =	$13\,440 e^{x^2} x^2 + 1680 e^{x^2} + 256 e^{x^2} x^8 + 3584 e^{x^2} x^6 + 13\,440 e^{x^2} x^4$
$f^{(9)}(x)$ =	$30\,240 e^{x^2} x + 512 e^{x^2} x^9 + 9216 e^{x^2} x^7 + 48\,384 e^{x^2} x^5 + 80\,640 e^{x^2} x^3$
$f^{(10)}(x)$ =	$302\,400 e^{x^2} x^2 + 30\,240 e^{x^2} + 1024 e^{x^2} x^{10} + 23\,040 e^{x^2} x^8 + 161\,280 e^{x^2} x^6 + 403\,200 e^{x^2} x^4$

f(x) = e^{x^2}

x–value 0

order of derivative

f(0.) =	1.
f′(0.) =	0.
f″(0.) =	2.
f‴(0.) =	0.
$f^{(4)}(0.)$ =	12.
$f^{(5)}(0.)$ =	0.
$f^{(6)}(0.)$ =	120.
$f^{(7)}(0.)$ =	0.
$f^{(8)}(0.)$ =	1680.

4.

f(x) = sec(x)

x–value x

order of derivative

$$f(x) = \sec(x)$$

$$f'(x) = \tan(x)\sec(x)$$

$$f''(x) = \sec^3(x) + \tan^2(x)\sec(x)$$

$$f'''(x) = 5\tan(x)\sec^3(x) + \tan^3(x)\sec(x)$$

$$f^{(4)}(x) = 5\sec^5(x) + 18\tan^2(x)\sec^3(x) + \tan^4(x)\sec(x)$$

$$f^{(5)}(x) = 61\tan(x)\sec^5(x) + 58\tan^3(x)\sec^3(x) + \tan^5(x)\sec(x)$$

$$f^{(6)}(x) = 61\sec^7(x) + 479\tan^2(x)\sec^5(x) + 179\tan^4(x)\sec^3(x) + \tan^6(x)\sec(x)$$

$$f^{(7)}(x) = 1385\tan(x)\sec^7(x) + 3111\tan^3(x)\sec^5(x) + 543\tan^5(x)\sec^3(x) + \tan^7(x)\sec(x$$

$$f^{(8)}(x) = 1385\sec^9(x) + 19\,028\tan^2(x)\sec^7(x) + 18\,270\tan^4(x)\sec^5(x) + 1636\tan^6(x)\sec^3(x) + \tan^8(x)\sec(x)$$

$$f^{(9)}(x) = 50\,521\tan(x)\sec^9(x) + 206\,276\tan^3(x)\sec^7(x) + 101\,166\tan^5(x)\sec^5(x) + 4916\tan^7(x)\sec^3(x) + \tan^9(x)\sec(x)$$

$$f^{(10)}(x) = 50\,521\sec^{11}(x) + 1\,073\,517\tan^2(x)\sec^9(x) + 1\,949\,762\tan^4(x)\sec^7(x) + 540\,242\tan^6(x)\sec^5(x) + 14\,757\tan^8(x)\sec^3(x) + \tan^{10}(x)\sec(x)$$

f(x) = sec(x)

x–value $\pi/4$

order of derivative

$$f(0.785398) = 1.41421$$

$$f'(0.785398) = 1.41421$$

$$f''(0.785398) = 4.24264$$

$$f'''(0.785398) = 15.5563$$

$$f^{(4)}(0.785398) = 80.6102$$

$$f^{(5)}(0.785398) = 510.531$$

5.

f(x) = tan(x)

x–value x

order of derivative

$f(x) =$	$\tan(x)$
$f'(x) =$	$\sec^2(x)$
$f''(x) =$	$2\tan(x)\sec^2(x)$
$f'''(x) =$	$2\sec^4(x) + 4\tan^2(x)\sec^2(x)$
$f^{(4)}(x) =$	$16\tan(x)\sec^4(x) + 8\tan^3(x)\sec^2(x)$
$f^{(5)}(x) =$	$16\sec^6(x) + 88\tan^2(x)\sec^4(x) + 16\tan^4(x)\sec^2(x)$
$f^{(6)}(x) =$	$272\tan(x)\sec^6(x) + 416\tan^3(x)\sec^4(x) + 32\tan^5(x)\sec^2(x)$
$f^{(7)}(x) =$	$272\sec^8(x) + 2880\tan^2(x)\sec^6(x) + 1824\tan^4(x)\sec^4(x) + 64\tan^6(x)\sec^2(x)$
$f^{(8)}(x) =$	$7936\tan(x)\sec^8(x) + 24\,576\tan^3(x)\sec^6(x) + 7680\tan^5(x)\sec^4(x) + 128\tan^7(x)\sec^2(x)$
$f^{(9)}(x) =$	$7936\sec^{10}(x) + 137\,216\tan^2(x)\sec^8(x) + 185\,856\tan^4(x)\sec^6(x) + 31\,616\tan^6(x)\sec^4(x) + 256\tan^8(x)\sec^2(x)$
$f^{(10)}(x) =$	$353\,792\tan(x)\sec^{10}(x) + 1\,841\,152\tan^3(x)\sec^8(x) + 1\,304\,832\tan^5(x)\sec^6(x) + 128\,512\tan^7(x)\sec^4(x) + 512\tan^9(x)\sec^2(x)$

f(x) = tan(x)

x–value $\pi/4$

order of derivative

$f(0.785398) =$	1.
$f'(0.785398) =$	2.
$f''(0.785398) =$	4.
$f'''(0.785398) =$	16.
$f^{(4)}(0.785398) =$	80.
$f^{(5)}(0.785398) =$	512.