What Rule?

a. $\int x e^{x^2}\, dx$ $\quad u = x^2 \quad du = 2x\,dx$

$\frac{1}{2} du = x\,dx$

$\int \frac{1}{2} e^u\, du =$

$\frac{1}{2} e^{x^2} + C$

b. $\int x \sin x\, dx$ $\quad u = x \quad dv = \sin x\, dx$

$du = dx \quad v = -\cos x\, dx$

$-x\cos x - \int -\cos x\, dx = -x\cos x + \sin x + C$

c. $\int \frac{x^2 - 4x + 7}{x^3 - x^2 + x + 3}\, dx$

$x^3 - x^2 + x + 3$ has a zero at $x = -1$ and so $(x+1)$ is a factor

$$\begin{array}{r|l} & x^2 - 2x + 3 \\ x+1 & x^3 - x^2 + x + 3 \\ & \underline{-x^3 \mp x^2} \\ & -2x^2 + x \\ & \underline{+2x^2 + 2x} \\ & 3x + 3 \\ & \underline{-3x \mp 3} \\ & 0 \end{array}$$

this is not factorable any further

$\int \frac{A}{x+1} + \frac{Bx + C}{x^2 - 2x + 3}\, dx$

$A(x^2 - 2x + 3) + (Bx + C)(x+1) = x^2 - 4x + 7$

if $x = -1$

$A(1 + 2 + 3) + 0 = 1 + 4 + 7$

$6A = 12$

$A = 2$

if $x = 0$

$2(3) + C(1) = 7$

$-6 \qquad -6$

$C = 1$

if $x = 1$

$2(1 - 2 + 3) + (B+1)(2) = 1 - 4 + 7$

$\cancel{4} + 2B + 2 = \cancel{4}$

$2B = -2$

$B = -1$

Thus,

$\int \frac{2}{x+1} + \frac{-1x + 1}{x^2 - 2x + 3}\, dx$

$= \int \frac{2}{x+1} - \frac{x - 1}{x^2 - 2x + 3}\, dx$

can this be done by substitution?

$u = x^2 - 2x + 3$

$du = 2x - 2\, dx$

$\frac{1}{2} du = (x-1)\,dx$

yes

$= 2\ln|x+1| - \frac{1}{2}\ln|x^2 - 2x + 3| + C$

d. $\int \cos^2\theta \, d\theta = \int \frac{1}{2}(1+\cos 2\theta)\, d\theta = \frac{1}{2}\left[\theta + \frac{1}{2}\sin 2\theta\right] + C$

e. $\int \frac{4}{x\sqrt{x^2-4}}\, dx = \frac{1}{2}\operatorname{arcsec}\left(\frac{x}{2}\right) + C$

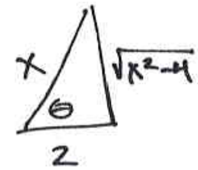
f. $\int \frac{1}{x^2\sqrt{x^2-4}}\, dx$ $\qquad x = 2\sec\theta \qquad \sqrt{x^2-4} = 2\tan\theta$

$x^2 = 4\sec^2\theta \qquad dx = 2\sec\theta\tan\theta\, d\theta$

$\int \frac{\cancel{2}\cancel{\sec\theta}\cancel{\tan\theta}\, d\theta}{4\sec^{\cancel{2}}\theta\, \cancel{2}\cancel{\tan\theta}} = \frac{1}{4}\int \frac{1}{\sec\theta}\, d\theta = \frac{1}{4}\int \cos\theta\, d\theta = +\frac{1}{4}\sin\theta + C$

$x = 2\sec\theta$

$\frac{x}{2} = \sec\theta = \frac{\text{hyp}}{\text{adj}}$



$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{\sqrt{x^2-4}}{x} \qquad \Rightarrow +\frac{1}{4}\sin\theta + C = \frac{\sqrt{x^2-4}}{4x} + C$

g. $\int \sqrt{\frac{3-x}{3+x}}\, dx$ depends on the formulas:

$$\int \sqrt{\frac{c+dx}{a+bx}}\, dx = \frac{c+dx}{|c+dx|}\left[\frac{\sqrt{(a+bx)(c+dx)}}{b} - \frac{ad-bc}{2b}\int \frac{dx}{\sqrt{(a+bx)(c+dx)}}\right]$$

and

$$\int \frac{dx}{\sqrt{(a+bx)(c+dx)}} = \begin{cases} \frac{2}{\sqrt{-bd}}\tan^{-1}\left(\frac{\sqrt{-bd(a+bx)(c+dx)}}{b(c+dx)}\right) \\ \text{or} \\ \frac{-1}{\sqrt{-bd}}\sin^{-1}\left(\frac{2bdx+ad+bc}{|ad-bc|}\right) & bd<0 \end{cases}$$

here $a=3 \quad c=3$

$b=1 \quad d=-1 \qquad bd = (1)(-1) = -1 < 0$

there is another formula for $bd > 0$

$ad-bc = 3(-1)-(1)(3) = -3-3 = -6$

now plug in

$$= \frac{3-x}{|3-x|}\left[\frac{\sqrt{9-x^2}}{1} + \frac{6}{2(1)}\cdot\frac{-1}{\sqrt{+1}}\sin^{-1}\left(\frac{-2x+0}{6}\right)\right] + C$$

$$\frac{3-x}{|3-x|}\left[\sqrt{9-x^2} + 3\sin\left(\frac{-x}{3}\right)\right] + C$$

The formulas often look worse than the final answers.