

### How do I know what integration rule to use?

Now that we've learned all these integration rules, how do we know which one to use?

Generally speaking, you want to consider the possibilities basically in the order we learned them.

For any integration problem, ask yourself:

- 1) Is this a basic rule?
- 2) Can I do some algebra on it (long division, etc.) to get it into the form of a basic rule?
- 3) Can I use u-substitution?
- 4) Can I use integration by parts?
- 5) Can I use a more "exotic" rule like partial fractions or trig substitution? (i.e. is it a rational function? does it have a square root in it?)
- 6) Does it look like something in the integration tables?
- 7) If all else fails, we are left with numerical integration or Taylor series (which we learn in Calc III).

Let's look at some examples.

a.  $\int xe^{x^2} dx$

Is this a basic rule? No. Can I do algebra on it? No. Can I do u-substitution? Yes.

b.  $\int x \ln x dx$

Is this a basic rule? No. Can I do algebra on it? No. Can I do u-sub? No. Can I use by parts? Yes.

c.  $\int \frac{x^2 - 4x + 7}{x^3 - x^2 + x + 3} dx$

Is this a basic rule? No. Can I do algebra on it? Well, I need to factor the denominator. But I can't do long division. Can I do u-sub? No. Can I use by parts? No. Since it's a rational function, can I use partial fractions? Yes.

d.  $\int \ln\left(\frac{x}{e^x}\right) dx$

Is this a basic rule? No. Can I do algebra on it? Yes, I can use log rules to split this and simplify it. Then, I want to reconsider each of the resulting integrals separately.

e.  $\int \frac{4}{x\sqrt{x^2 - 4}} dx$

Is this a basic rule? If we were in the version of calculus that used trig functions, we could do this, but since we haven't learned those rules, we'd answer no, and move on. Can I do u-sub? No. Can I do by-parts? It won't actually help since we don't have the x in the numerator. We should try tables.

f.  $\int \frac{1}{x^2 \sqrt{x^2 - 4}} dx$

Is this a basic rule? No. Can I do algebra on it? No. Can I do u-sub? No. Can I do by parts? No. We should check the tables.

g.  $\int \sqrt{\frac{3-x}{3+x}} dx$

Is this a basic rule? No. Can I do algebra on it? Not in a way that will help. Can I do u-sub? No. Can I do by parts? Not in a way that will help easily. Can I do partial fractions or trig on it? No. Can I use integration by table? Yes, if the text I have includes this rule. Otherwise, we might try simplifying the expression (possibly by long division). Or get a bigger table.

**Problems.**

h. Do each of the problems above.

i.  $\int \frac{5x-12}{x^2-4} dx$

j.  $\int \frac{x^3}{\sqrt{4+x^2}} dx$

k.  $\int_0^1 \frac{x^2-x}{x^2+x+1} dx$

l.  $\int \frac{xe^x}{(x+1)^2} dx$

m.  $\int \frac{x^2}{\sqrt[3]{2x+1}} dx$

n.  $\int x^2 e^{x^3} dx$

o.  $\int \frac{x^2+4}{x-1} dx$

p.  $\int \sqrt{4x^2+1} dx$