Implicit Key

1. $3x^2 - xy + xz - 2yz^3 + z^4 = \frac{z}{x}$ need $\frac{\partial z}{\partial y}$

$-x + xz_y - 2z^3 - 2y \cdot 3z^2 z_y + 4z^3 z_y = \frac{1}{x} \cdot z_y$

$z_y(x - 6yz^2 + 4z^3 - \frac{1}{x}) = x + 2z^3$

$z_y = \dfrac{x + 2z^3}{x - 6yz^2 + 4z^3 - \frac{1}{x}}$ $\quad \frac{x}{x} \quad =$

$\dfrac{x^2 + 2z^3x}{x^2 - 6yz^2x + 4z^3x - 1}$

6. $F(x,y,z) = 3x^2 - xy + xz - 2yz^3 + z^4 - \frac{z}{x} = 0$

$-\dfrac{F_y}{F_z} = -\dfrac{-x - 2z^3}{x - 6yz^2 + 4z^3 - \frac{1}{x}} = \dfrac{x + 2z^3}{x - 6yz^2 + 4z^3 - \frac{1}{x}}$

2. find $\frac{dy}{dx}$ $\quad 4x^3 - \sin(xy) = ye^x$

$12x^2 - \cos(xy)[y + xy'] = y'e^x + ye^x$

$12x^2 - y\cos xy - xy'\cos xy = y'e^x + ye^x$

$12x^2 - y\cos xy - ye^x = y'e^x + xy'\cos xy$

$12x^2 - y\cos xy - ye^x = y'(e^x + x\cos xy)$

$\dfrac{12x^2 - y\cos xy - ye^x}{e^x + x\cos xy} = y'$

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7. $F(x,y) = 4x^3 - \sin(xy) - ye^x = 0$

$$-\frac{F_x}{F_y} = -\frac{12x^2 - \cos(xy)\cdot y - ye^x}{-\cos(xy)\cdot x - e^x}$$

$$= \frac{12x^2 - y\cos(xy) - ye^x}{x\cos(xy) + e^x} \checkmark$$

3. find $\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}$ $\quad x^5 + xyz + z^4 = \ln\left(\frac{x}{y}\right) = \ln(x) - \ln(y)$

$\frac{\partial z}{\partial x}$: $5x^4 + yz + xyz_x + 4z^3 z_x = \frac{1}{x}$

$$z_x(xy + 4z^3) = \frac{1}{x} - 5x^4 + yz$$

$$z_x = \frac{\frac{1}{x} - 5x^4 + yz}{xy + 4z^3} = \frac{1 - 5x^5 + xyz}{x^2y + 4xz^3}$$

$\frac{\partial z}{\partial y}$: $xz + xyz_y + 4z^3 z_y = -\frac{1}{y}$

$$z_y(xy + 4z^3) = \frac{-1}{y} - xz$$

$$z_y = \frac{-\frac{1}{y} - xz}{xy + 4z^3} = \frac{-1 - xyz}{xy^2 + 4yz^3}$$

8. $F(x,y,z) = x^5 + xyz + z^4 - \ln x + \ln y = 0$

$$-\frac{F_x}{F_z} = -\frac{5x^4 + yz - \frac{1}{x}}{xy + 4z^3} = \frac{\frac{1}{x} - 5x^4 - yz}{xy + 4z^3} \checkmark$$

$$-\frac{F_y}{F_z} = -\frac{xz + \frac{1}{y}}{xy + 4z^3} = \frac{-xz - \frac{1}{y}}{xy + 4z^3} \checkmark$$

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4. find $\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}$ $\quad x^2 - 2y^2z^3 + z = (1+xe^{y^2}e^{z^2})^{1/2}(xyz)^{-1}$

$\frac{\partial z}{\partial x}$: $2x - 6y^2z^2 z_x + z_x =$
$\frac{1}{2}(1+xe^{y^2}e^{z^2})^{-1/2}(e^{y^2}e^{z^2} + xe^{y^2}e^{z^2}\cdot 2z z_x)(xyz)^{-1}$
$+ (1+xe^{y^2}e^{z^2})^{1/2}(xyz)^{-2}(yz + xy z_x)(-1)$

$$2x - 6y^2z^2z_x + z_x = \frac{e^{y^2}e^{z^2} + xe^{y^2}e^{z^2}2z z_x}{2xyz(1+xe^{y^2}e^{z^2})^{1/2}} + \frac{-yz - xyz_x}{(xyz)^2}(1+xe^{y^2}e^{z^2})^{1/2}$$

$$2x - \frac{e^{y^2}e^{z^2}}{2xyz(1+xe^{y^2}e^{z^2})^{1/2}} + \frac{\cancel{yz}(1+xe^{y^2}e^{z^2})^{1/2}}{x^2y^{\cancel{2}}z^{\cancel{2}}} =$$

$$z_x\left(6y^2z^2 - 1 + \frac{xe^{y^2}e^{z^2}\cdot 2z}{2xyz(1+xe^{y^2}e^{z^2})^{1/2}} - \frac{xy(1+xe^{y^2}e^{z^2})^{1/2}}{x^2y^2z^2}\right)$$

$$z_x = \frac{2x - \dfrac{e^{y^2}e^{z^2}}{2xyz(1+xe^{y^2}e^{z^2})^{1/2}} + \dfrac{(1+xe^{y^2}e^{z^2})^{1/2}}{x^2yz}}{6y^2z^2 - 1 + \dfrac{e^{y^2}e^{z^2}}{y(1+xe^{y^2}e^{z^2})^{1/2}} - \dfrac{(1+xe^{y^2}e^{z^2})^{1/2}}{xyz^2}}$$

$\frac{\partial z}{\partial y}$: $-4yz^3 - 6y^2z^2z_y + z_y = \frac{1}{2}(1+xe^{y^2}e^{z^2})^{-1/2}(x\cdot 2ye^{y^2}e^{z^2} + xe^{y^2}\cdot 2ze^{z^2} z_y)$
$(xyz)^{-1} + (1+xe^{y^2}e^{z^2})^{1/2}(xyz)^{-2}(xz + xyz_y)(-1)$

$$-4yz^3 - 6y^2z^2z_y + z_y = \frac{2xye^{y^2}e^{z^2} + 2xze^{y^2}e^{z^2}z_y}{2xyz(1+xe^{y^2}e^{z^2})^{1/2}} - \frac{(xz + xyz_y)\sqrt{\ }}{x^2y^2z^2}$$

$$-4yz^3 - \frac{\cancel{2xy}e^{y^2}e^{z^2}}{\cancel{2xy}z(1+xe^{y^2}e^{z^2})^{1/2}} + \frac{\cancel{xz}(1+xe^{y^2}e^{z^2})^{1/2}}{x^{\cancel{2}}y^2z^{\cancel{2}}} =$$

$$z_y\left(6y^2z^2 - 1 + \frac{2xze^{y^2}e^{z^2}}{2xyz(1+xe^{y^2}e^{z^2})^{1/2}} - \frac{xy(1+xe^{y^2}e^{z^2})^{1/2}}{x^2y^2z^2}\right)$$

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$$z_y = \frac{-4yz^3 - \frac{e^{y^2}e^{z^2}}{(1+xe^{y^2}e^{z^2})^{1/2}} + \frac{(1+xe^{y^2}e^{z^2})^{1/2}}{xy^2z}}{6y^2z^2-1 + \frac{e^{y^2}e^{z^2}}{y(1+xe^{y^2}e^{z^2})^{1/2}} - \frac{(1+xe^{y^2}e^{z^2})^{1/2}}{xyz^2}}$$

9. $F(x,y,z) = x^2 - 2y^2z^3 + z - (1+xe^{y^2}e^{z^2})^{1/2}(xyz)^{-1}$

$$-\frac{F_x}{F_z} = -\frac{2x - \frac{1}{2}(1+xe^{y^2}e^{z^2})^{-1/2}e^{y^2}e^{z^2}/xyz + (1+xe^{y^2}e^{z^2})^{1/2}\frac{1}{x^2yz}}{-6y^2z^2+1-\frac{1}{\cancel{2}}(1+xe^{y^2}e^{z^2})^{-1/2}\frac{(xe^{y^2}e^{z^2}\cancel{2z})}{xy\cancel{z}} + \frac{(1+xe^{y^2}e^{z^2})^{1/2}}{xyz^2}}$$

$$-\frac{F_y}{F_z} = -\frac{-4y^2z^3 - \frac{1}{\cancel{2}}(1+xe^{y^2}e^{z^2})^{-1/2}\,x\,e^{y^2}e^{z^2}\cdot 2y/xyz - (1+xe^{y^2}e^{z^2})^{1/2}\frac{1}{xy^2z}}{-6y^2z^2+1-\frac{\frac{1}{\cancel{2}}(1+xe^{y^2}e^{z^2})^{-1/2}}{xy\cancel{z}}\cdot\cancel{2z}\,e^{y^2}e^{z^2} + \frac{(1+xe^{y^2}e^{z^2})^{1/2}}{xyz^2}}$$

5. find $\frac{\partial w}{\partial x}, \frac{\partial w}{\partial y}, \frac{\partial w}{\partial z}$ $\quad 2x^2w - xyz - 2w\tan(zw) + e^{4w} = \frac{zy}{w}$
$= zyw^{-1}$

$\frac{\partial w}{\partial x}$: $4xw + 2x^2w_x - yz - 2w_x\tan(zw) - 2w\sec^2(zw)\cdot zw_x$
$+ e^{4w}\cdot 4w_x = -zyw^{-2}w_x$

$4wx - yz = \left(-2x^2 + 2\tan(zw) + 2w\sec^2(zw)\cdot z - 4e^{4w} - \frac{zy}{w^2}\right)w_x$

$$w_x = \frac{4wx - yz}{-2x^2 + 2\tan(zw) + 2w\sec^2(zw)\cdot z - 4e^{4w} - \frac{zy}{w^2}}$$

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$\frac{\partial w}{\partial y}$: $2x^2 w_y - xz - 2w_y \tan(zw) - 2w\sec^2(zw)zw_y$
$+4w_y e^{4w} = \frac{z}{w} - \frac{zy}{w^2} w_y$

$-zx - \frac{z}{w} = \left(-2x^2 + 2\tan(zw) + 2w\sec^2(zw)z - 4e^{4w} - \frac{zy}{w^2}\right) w_y$

$$w_y = \frac{-zx - \frac{z}{w}}{-2x^2 + 2\tan(zw) + 2w\sec^2(zw)z - 4e^{4w} - \frac{zy}{w^2}}$$

$\frac{\partial w}{\partial z}$: $2x^2 w_z - xy - 2w_z \tan(zw) - 2w\sec^2(zw)(w + zw_z)$
$+ 4w_z e^{4w} = yw^{-1} - \frac{zy}{w^2} w_z$

$-xy - 2w^2\sec^2(zw) - \frac{y}{w} = w_z\left(-2x^2 + 2\tan(zw) + 2wz\sec^2(wz)\right.$
$\left. - 4e^{4w} - \frac{zy}{w^2}\right)$

$$w_z = \frac{-xy - 2w^2\sec^2(zw) - \frac{y}{w}}{-2x^2 + 2\tan(zw) + 2wz\sec^2(wz) - 4e^{4w} - \frac{zy}{w^2}}$$

10. $F(x, y, z, w) = 2x^2 w - xyz - 2w\tan(zw) + e^{4w} - \frac{zy}{w} = 0$

$$-\frac{F_x}{F_w} = -\frac{4xw - yz}{2x^2 - 2\tan(zw) - 2w\sec^2(zw)\cdot z + 4e^{4w} + \frac{zy}{w^2}}$$

$$-\frac{F_y}{F_w} = -\frac{-xz - \frac{z}{w}}{2x^2 - 2\tan(zw) - 2w\sec^2(zw) + 4e^{4w} + \frac{zy}{w^2}}$$

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$$-\frac{F_z}{F_w} = -\frac{-xy - 2w\sec^2(zw)\cdot w - \frac{y}{w}}{2x^2 - 2\tan(zw) - 2w\sec^2(zw) + 4e^{4w} + \frac{zy}{w^2}}$$