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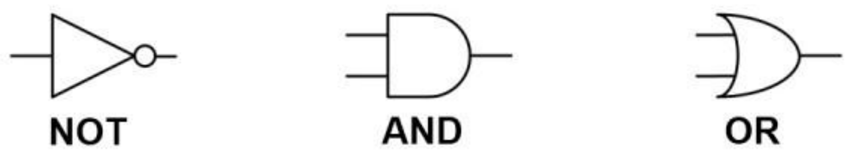
Logic Gates & Circuits

Logic gates or logic circuits are a way of representing truth values of propositions and operators or Boolean algebra. Since truth values and truth values are interchangeable, essentially the same rules apply to both types. The only transformation that needs to take place is that Truth=1 and False=0. There are a small number of such logic gates and certain modifications of them.

The three primary operators are:



Other combinations can be built from these. For instance, if you add the small circle to either the AND or OR gate, you get the NAND operator (the sum followed by negation), and the NOR operator (or followed by negation).

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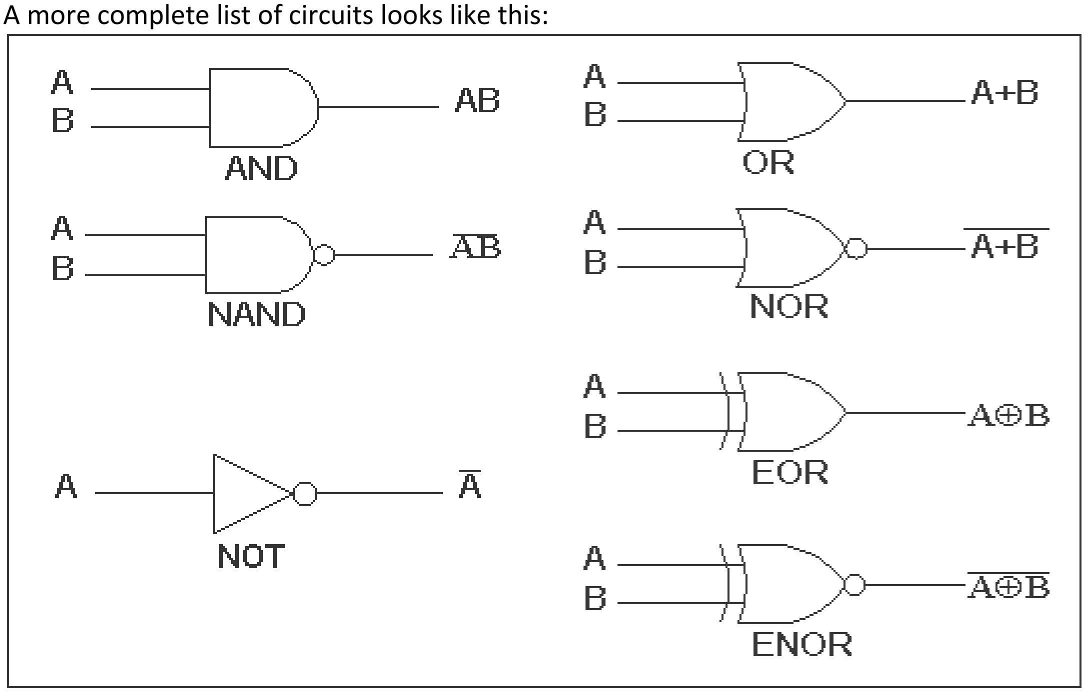
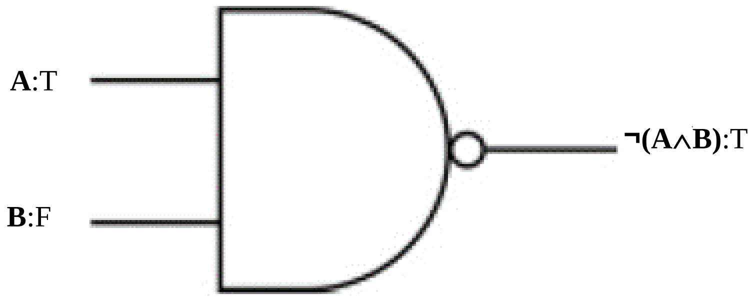
Adding a small half-arc in front of the operator can also create new operators. Added to the OR operator it produces XOR (exclusive or).

A more complete list of circuits looks like this:



We can analyze the truth values of circuits the same way that we build truth tables.

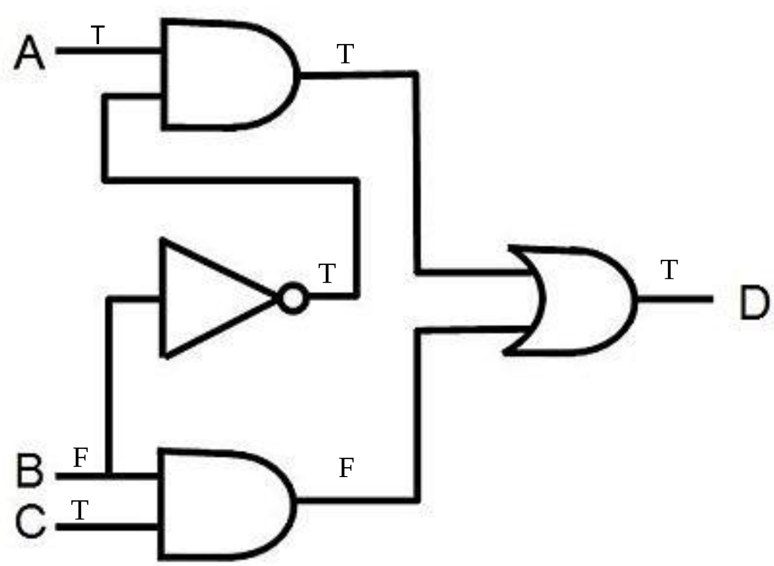
Example 1. Suppose that the circuit shown below. If the input for A is truth and the input for B is false, what is the output of the circuit?



The circuit displayed is NAND (the negation of the AND operator). Since A is True and B is False, the AND operator produces A∧B is False since one of them is true. And then we negate this, to obtain True for the NAND operator.

Let's look at a more complex example.

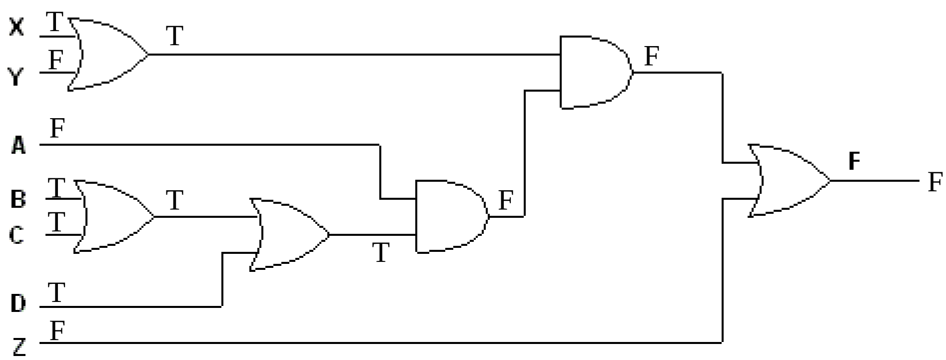
Example 2. Find the truth value of D in the circuit shown below. Suppose that A has an initial truth value of "True", B has a truth value of "False" and C has a truth value of "True".



By following the logical operators, we can determine that the truth value of the compound statement D is "True".

The statement D in logical operator notation represents the compound statement (A∧¬B)∨(B∧C).

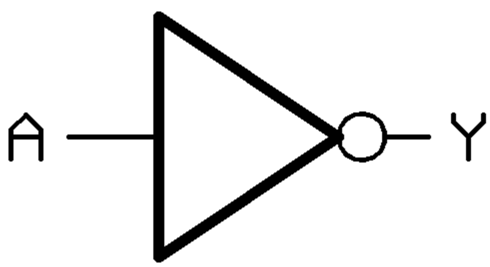
Example 3. Find the truth value of the compound statement F if X, B, C and D are true, and Y, Z and A are false. Then write the compound statement with logical operators.



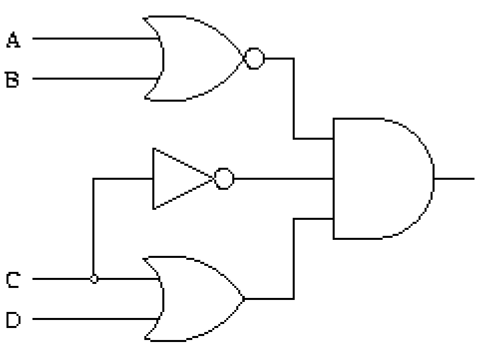
The compound statement can be written as ((X∨Y)∧(A∧(D∨(B∨C))))∨Z.

Practice Problems.

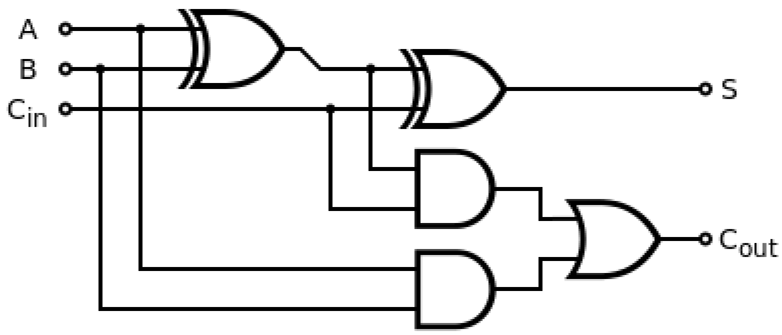
1. Find the value of the logical circuit below if each of the input values is A is true.



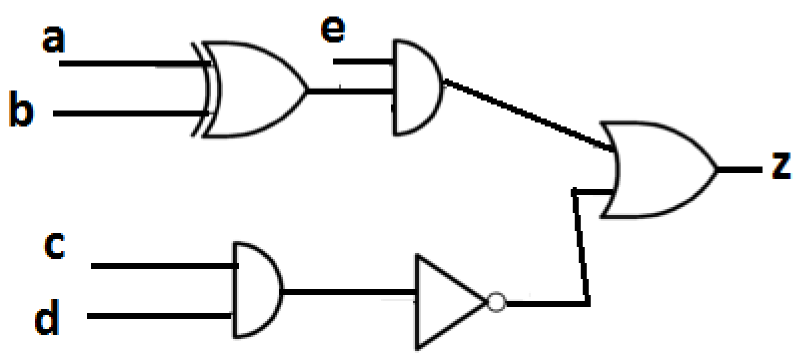
2. Find the value of the logical circuit below if B and C are true and A and D are false.



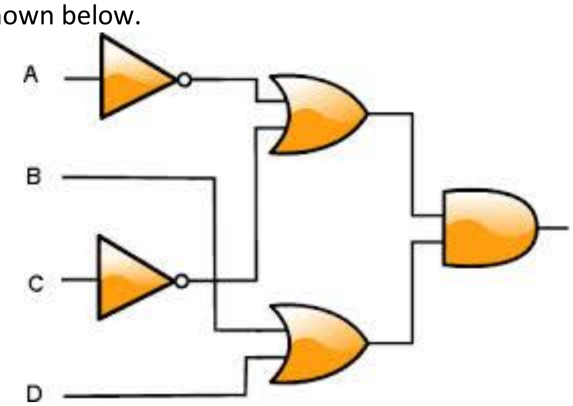
3. Find the value of the outputs S and C_{out} if the inputs are A and C_{in} are true, and B is false.



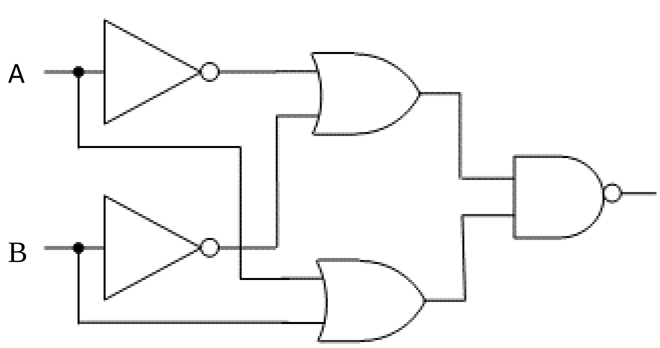
4. Find the value of the logical circuit's output if A, D and E are true, and B and C are false.



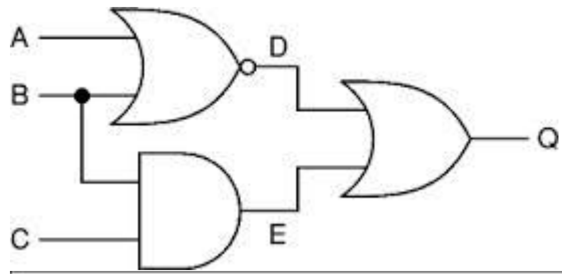
5. Write the compound statement using logical operators that is equivalent to the logic circuits shown below.



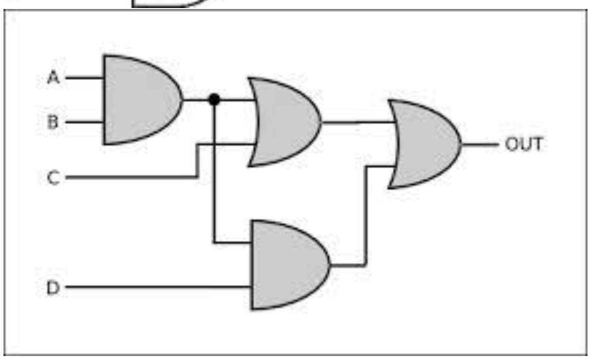
a.



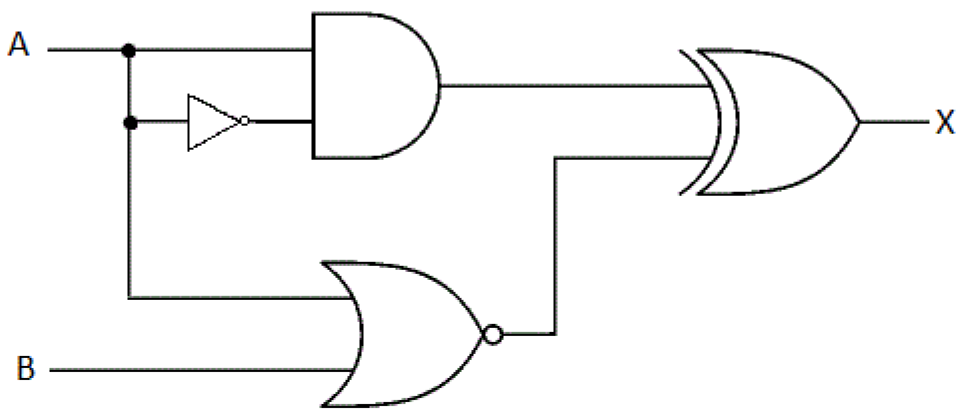
b.



c.



d.



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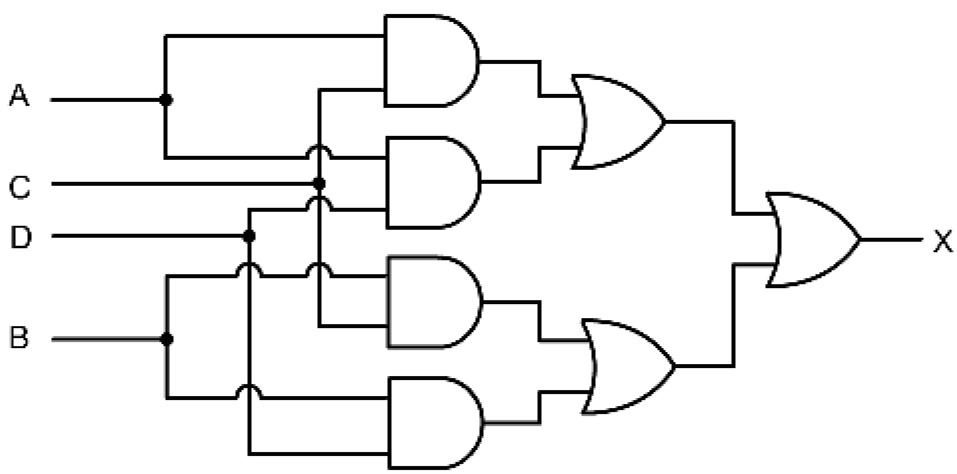
e.



f.

6. Represent each of the following expressions written with logical operators in the form of logic gates.
   a. $A \lor \neg B$

b. $((B \wedge C) \vee \neg A) \wedge \neg B$
c. $\neg(B \wedge (A \vee B)) \wedge (D \oplus (A \wedge \neg C))$
d. $(((X \vee Y) \wedge \neg(A \vee D)) \oplus (B \vee (\neg A \wedge C))) \wedge A$

7. Rewrite each of the problems above in Boolean Algebra notation with 1 and 0 instead of True and False, and using addition, multiplication and bars instead of $\vee$, $\wedge$ and $\neg$ respectively.