Math 2568 Discrete Dynamical Systems Key

1a. $A = \begin{bmatrix} .38 & .24 \\ -.36 & 1.22 \end{bmatrix}$ $\vec{x}_0 = \begin{bmatrix} 15 \\ 20 \end{bmatrix}$

x_i: $\begin{bmatrix} 10.5 \\ 19 \end{bmatrix}, \begin{bmatrix} 8.55 \\ 19.4 \end{bmatrix}, \begin{bmatrix} 7.905 \\ 20.59 \end{bmatrix}, \begin{bmatrix} 7.9455 \\ 22.274 \end{bmatrix}, \begin{bmatrix} 8.365 \\ 24.31 \end{bmatrix}, \begin{bmatrix} 9.01 \\ 26.65 \end{bmatrix} \begin{bmatrix} 9.82 \\ 29.27 \end{bmatrix}, \begin{bmatrix} 10.76 \\ 32.17 \end{bmatrix}$

$\begin{bmatrix} 11.81 \\ 35.38 \end{bmatrix} \begin{bmatrix} 12.98 \\ 38.91 \end{bmatrix} \begin{bmatrix} 14.27 \\ 42.8 \end{bmatrix} \begin{bmatrix} 15.69 \\ 47.08 \end{bmatrix}, \begin{bmatrix} 17.26 \\ 51.78 \end{bmatrix} \begin{bmatrix} 18.99 \\ 56.96 \end{bmatrix} \begin{bmatrix} 20.87 \\ 62.66 \end{bmatrix}$

This is probably a saddle point (values reduced before growing again)

graphs attached on a separate page

b. $A = \begin{bmatrix} 1 & -1 \\ 2 & 4 \end{bmatrix}$ $x_0 = \begin{bmatrix} 2 \\ 5 \end{bmatrix}$

$x_i = \begin{bmatrix} -3 \\ 24 \end{bmatrix}, \begin{bmatrix} -27 \\ 90 \end{bmatrix} \begin{bmatrix} -117 \\ 306 \end{bmatrix}, \begin{bmatrix} -423 \\ 990 \end{bmatrix}, \begin{bmatrix} -1413 \\ 3114 \end{bmatrix}, \begin{bmatrix} -4527 \\ 9630 \end{bmatrix} \begin{bmatrix} -14157 \\ 29466 \end{bmatrix}, \begin{bmatrix} -43623 \\ 89550 \end{bmatrix},$

$\begin{bmatrix} -133,173 \\ 270,954 \end{bmatrix} \begin{bmatrix} -404,127 \\ 817,470 \end{bmatrix} \begin{bmatrix} -1,221,597 \\ 2,461,626 \end{bmatrix}, \begin{bmatrix} -3,683,223 \\ 7,403,310 \end{bmatrix}, \begin{bmatrix} -11,086,533 \\ 22,246,794 \end{bmatrix}, \begin{bmatrix} -33,333,327 \\ 66,814,110 \end{bmatrix}$

repeller.

c. $A = \begin{bmatrix} 37/21 & 10/21 \\ 15/21 & 12/21 \end{bmatrix}$ $x_0 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$x_i = \begin{bmatrix} 2.71 \\ 1.86 \end{bmatrix}, \begin{bmatrix} 5.67 \\ 3 \end{bmatrix}, \begin{bmatrix} 11.41 \\ 5.76 \end{bmatrix}, \begin{bmatrix} 22.85 \\ 11.44 \end{bmatrix}, \begin{bmatrix} 45.71 \\ 22.86 \end{bmatrix}, \begin{bmatrix} 91.43 \\ 45.72 \end{bmatrix} \begin{bmatrix} 182.86 \\ 91.43 \end{bmatrix}, \begin{bmatrix} 365.71 \\ 182.86 \end{bmatrix}, \begin{bmatrix} 731.43 \\ 365.71 \end{bmatrix}$

$\begin{bmatrix} 1462.9 \\ 731.43 \end{bmatrix}, \begin{bmatrix} 2925.7 \\ 1462.9 \end{bmatrix}, \begin{bmatrix} 5851.4 \\ 2925.7 \end{bmatrix} \begin{bmatrix} 11,702.9 \\ 5851.4 \end{bmatrix} \begin{bmatrix} 23,405.7 \\ 11,702.9 \end{bmatrix}, \begin{bmatrix} 46,811.4 \\ 23405.7 \end{bmatrix}$

Saddle point

d. $A = \begin{bmatrix} 2 & 0 \\ 0 & .5 \end{bmatrix}$ $x_0 = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$ This matrix is triangular, so we can read eigenvalues off diagonal $\Rightarrow$ saddle point

$x_i = \begin{bmatrix} 10 \\ 1.5 \end{bmatrix}, \begin{bmatrix} 20 \\ .75 \end{bmatrix}, \begin{bmatrix} 40 \\ .375 \end{bmatrix}, \begin{bmatrix} 80 \\ 1875 \end{bmatrix} \begin{bmatrix} 160 \\ .094 \end{bmatrix} \begin{bmatrix} 320 \\ .047 \end{bmatrix} \begin{bmatrix} 640 \\ .023 \end{bmatrix} \begin{bmatrix} 1280 \\ .012 \end{bmatrix} \begin{bmatrix} 2560 \\ .0058 \end{bmatrix}, \begin{bmatrix} 5120 \\ .0029 \end{bmatrix}$

$\begin{bmatrix} 10,240 \\ .0015 \end{bmatrix} \begin{bmatrix} 20,480 \\ 7 \times 10^{-4} \end{bmatrix} \begin{bmatrix} 40960 \\ 3.6 \times 10^{-4} \end{bmatrix} \begin{bmatrix} 81,920 \\ 1.8 \times 10^{-4} \end{bmatrix} \begin{bmatrix} 163,840 \\ 9.2 \times 10^{-5} \end{bmatrix}$

1e. $A = \begin{bmatrix} 1 & .5 \\ 1 & 1.5 \end{bmatrix}$ $x_0 = \begin{bmatrix} -2 \\ 0 \end{bmatrix}$

$x_i = \begin{bmatrix} 3 \\ 3 \end{bmatrix} \begin{bmatrix} 9.5 \\ 22.5 \end{bmatrix} \begin{bmatrix} 20.75 \\ 43.25 \end{bmatrix} \begin{bmatrix} 42.375 \\ 85.63 \end{bmatrix} \begin{bmatrix} 85.19 \\ 170.81 \end{bmatrix} \begin{bmatrix} 170.59 \\ 341.41 \end{bmatrix} \begin{bmatrix} 341.3 \\ 682.7 \end{bmatrix} \begin{bmatrix} 682.6 \\ 1365.4 \end{bmatrix},$

$\begin{bmatrix} 1365.3 \\ 2730.7 \end{bmatrix} \begin{bmatrix} 2730.7 \\ 5461.3 \end{bmatrix} \begin{bmatrix} 5461.3 \\ 10,922.7 \end{bmatrix}, \begin{bmatrix} 10,922.7 \\ 21845.3 \end{bmatrix} \begin{bmatrix} 21845.3 \\ 43,690.7 \end{bmatrix} \begin{bmatrix} 43,690.7 \\ 87,381.3 \end{bmatrix}, \begin{bmatrix} 87,381.3 \\ 174,762.7 \end{bmatrix}$

probably a repeller (but could be a saddle point?)

f. $A = \begin{bmatrix} 1.71 & -.707 \\ 1 & 0 \end{bmatrix}$ $x_0 = \begin{bmatrix} 11 \\ 13 \end{bmatrix}$

$x_i = \begin{bmatrix} 9.619 \\ 11 \end{bmatrix}, \begin{bmatrix} 8.167 \\ 9.62 \end{bmatrix} \begin{bmatrix} 8.03 \\ 8.67 \end{bmatrix}, \begin{bmatrix} 7.60 \\ 8.03 \end{bmatrix} \begin{bmatrix} 7.31 \\ 7.60 \end{bmatrix} \begin{bmatrix} 7.14 \\ 7.31 \end{bmatrix}, \begin{bmatrix} 7.03 \\ 7.14 \end{bmatrix}, \begin{bmatrix} 6.98 \\ 7.03 \end{bmatrix}$

$\begin{bmatrix} 6.96 \\ 6.98 \end{bmatrix} \begin{bmatrix} 6.97 \\ 6.96 \end{bmatrix} \begin{bmatrix} 7.00 \\ 6.97 \end{bmatrix} \begin{bmatrix} 7.04 \\ 7.00 \end{bmatrix} \begin{bmatrix} 7.09 \\ 7.04 \end{bmatrix} \begin{bmatrix} 7.15 \\ 7.09 \end{bmatrix} \begin{bmatrix} 7.21 \\ 7.15 \end{bmatrix} \begin{bmatrix} 7.27 \\ 7.21 \end{bmatrix}$

Saddle point

g. $A = \begin{bmatrix} 1.8 & -.81 \\ 1 & 0 \end{bmatrix}$ $x_0 = \begin{bmatrix} 15 \\ 3 \end{bmatrix}$

$x_i = \begin{bmatrix} 24.57 \\ 15 \end{bmatrix}, \begin{bmatrix} 32.08 \\ 24.57 \end{bmatrix}, \begin{bmatrix} 37.84 \\ 32.08 \end{bmatrix} \begin{bmatrix} 42.12 \\ 37.84 \end{bmatrix}, \begin{bmatrix} 45.17 \\ 42.12 \end{bmatrix}, \begin{bmatrix} 47.197 \\ 45.17 \end{bmatrix}, \begin{bmatrix} 48.36 \\ 47.19 \end{bmatrix}, \begin{bmatrix} 48.81 \\ 48.36 \end{bmatrix},$

$\begin{bmatrix} 48.69 \\ 48.81 \end{bmatrix}, \begin{bmatrix} 48.11 \\ 48.69 \end{bmatrix}, \begin{bmatrix} 47.17 \\ 48.12 \end{bmatrix}, \begin{bmatrix} 45.92 \\ 47.17 \end{bmatrix}, \begin{bmatrix} 44.46 \\ 45.92 \end{bmatrix}, \begin{bmatrix} 42.83 \\ 44.46 \end{bmatrix}, \begin{bmatrix} 41.08 \\ 42.86 \end{bmatrix}$

attractor? (it grows, but then shrinks).

h. $A = \begin{bmatrix} 1.24 & -.97 \\ 1 & 0 \end{bmatrix}$ $x_0 = \begin{bmatrix} -2 \\ 12 \end{bmatrix}$

$x_i = \begin{bmatrix} -14.12 \\ -2 \end{bmatrix}, \begin{bmatrix} -15.56 \\ -14.12 \end{bmatrix}, \begin{bmatrix} -5.61 \\ -15.57 \end{bmatrix}, \begin{bmatrix} 8.14 \\ -5.61 \end{bmatrix}, \begin{bmatrix} 15.54 \\ 8.15 \end{bmatrix}, \begin{bmatrix} 11.37 \\ 15.54 \end{bmatrix}, \begin{bmatrix} -.98 \\ 11.37 \end{bmatrix}, \begin{bmatrix} -12.24 \\ -.98 \end{bmatrix},$

$\begin{bmatrix} -14.23 \\ -12.24 \end{bmatrix}, \begin{bmatrix} -5.77 \\ -14.23 \end{bmatrix}, \begin{bmatrix} 6.64 \\ -5.77 \end{bmatrix}, \begin{bmatrix} 13.84 \\ 6.64 \end{bmatrix}, \begin{bmatrix} 10.71 \\ 13.84 \end{bmatrix}, \begin{bmatrix} -.13 \\ 10.71 \end{bmatrix}, \begin{bmatrix} -10.56 \\ -.13 \end{bmatrix}, \begin{bmatrix} -12.96 \\ -10.56 \end{bmatrix}$

Rotation

1. i. $A = \begin{bmatrix} 1.24 & -1.03 \\ 1 & 0 \end{bmatrix}$ $x_0 = \begin{bmatrix} -5 \\ -8 \end{bmatrix}$

$x_i = \begin{bmatrix} 2.04 \\ -5 \end{bmatrix}, \begin{bmatrix} 7.68 \\ 2.04 \end{bmatrix}, \begin{bmatrix} 7.42 \\ 7.68 \end{bmatrix}, \begin{bmatrix} 1.29 \\ 7.42 \end{bmatrix}, \begin{bmatrix} -6.04 \\ 1.29 \end{bmatrix}, \begin{bmatrix} -8.82 \\ -6.04 \end{bmatrix}, \begin{bmatrix} -4.72 \\ 8.82 \end{bmatrix}, \begin{bmatrix} 3.24 \\ -4.72 \end{bmatrix},$

$\begin{bmatrix} 8.87 \\ 3.23 \end{bmatrix}, \begin{bmatrix} 7.67 \\ 8.87 \end{bmatrix}, \begin{bmatrix} .37 \\ 7.67 \end{bmatrix}, \begin{bmatrix} -7.44 \\ .37 \end{bmatrix}, \begin{bmatrix} -9.61 \\ -7.44 \end{bmatrix}, \begin{bmatrix} -4.24 \\ -9.61 \end{bmatrix}, \begin{bmatrix} 4.62 \\ -4.24 \end{bmatrix}$

rotation

2a. $A = \begin{bmatrix} .978 & -.006 \\ .004 & .992 \end{bmatrix}$ $(.978-\lambda)(.992-\lambda) + (.004)(.006) = 0$

Foil or use Calculator

min at $(.985, -2.5E-5)$

$\lambda = .99, .98$ The origin will be an attractor

b. $A = \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$ $(2-\lambda)(1-\lambda) - 0 = 0$ $\lambda = 1, \lambda = 2$

repeller

c. $A = \begin{bmatrix} 4 & -2 \\ 1 & 1 \end{bmatrix}$

$(4-\lambda)(1-\lambda) + 2 = 0$

$4 - 4\lambda - \lambda + \lambda^2 + 2 = \lambda^2 - 5\lambda + 6 = 0$

$(\lambda - 2)(\lambda - 3) = 0$ $\lambda = 2, 3$ repeller

d. $A = \begin{bmatrix} .4 & .3 \\ -.32 & 1.2 \end{bmatrix}$

$(.4-\lambda)(1.2-\lambda) + .3*(.32) = 0$

$\lambda = .547$ $\lambda = 1.053$

Foil or use calculator

Saddlepoint

Check of problems in 1.a-i for behaviour.

b. $\begin{bmatrix} 1 & -1 \\ 2 & 4 \end{bmatrix} \Rightarrow (1-\lambda)(4-\lambda)+2=0$

$4-\lambda-4\lambda+\lambda^2+2=\lambda^2-5\lambda+6=0 \quad \lambda=2,3$

repeller.

a. $\begin{bmatrix} .38 & .24 \\ -.36 & 1.22 \end{bmatrix}$ $(.38-\lambda)(1.22-\lambda)+.24(.36)=0$

$\lambda=.5 \quad \lambda=1.1.$ Saddle point

c. $\begin{bmatrix} 37/21 & 10/21 \\ 15/21 & 12/21 \end{bmatrix}$ $(37/21-\lambda)(12/21-\lambda)-\frac{10\cdot 15}{21\cdot 21}=0$

$\lambda=1/3 \quad \lambda=2$

Saddle point (we started close to eigenvector)

d. saddle point $\lambda=2, \lambda=.5$

e. $\begin{bmatrix} 1 & .5 \\ 1 & 1.5 \end{bmatrix}$ $(1-\lambda)(1.5-\lambda)-.5=0$

$\lambda=.5 \quad \lambda=2$

Saddle point

f. $\begin{bmatrix} 1.71 & -.707 \\ 1 & 0 \end{bmatrix}$ $(1.71-\lambda)(-\lambda)+.707=0$

$\lambda=.7 \quad \lambda=1.19$

Saddle point

g. $\begin{bmatrix} 1.8 & -.81 \\ 1 & 0 \end{bmatrix}$ $(1.8-\lambda)(-\lambda)+.81=0$

$\lambda=.9$ repeated attractor

h. $\begin{bmatrix} 1.24 & -.97 \\ 1 & 0 \end{bmatrix}$ $(1.24-\lambda)(-\lambda)+.97=0$

no real solution

$-1.24\lambda+\lambda^2+.97=\lambda^2-1.24\lambda+.97=0$

$\lambda=\frac{1.24\pm\sqrt{1.24^2-4(1)(.97)}}{2} \approx .62\pm.765i$

$\sqrt{.62^2+.765^2} \approx .9846$ rotation, w/ very slow attraction

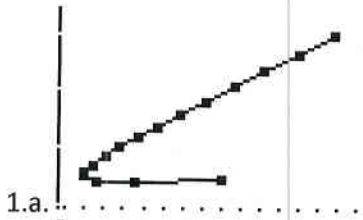
L.O. $\begin{bmatrix} 1.24 & -1.03 \\ 1 & 0 \end{bmatrix}$ $(1.24-\lambda)(-\lambda)+1.03=0$

$\lambda^2 - 1.24\lambda + 1.03 = 0$

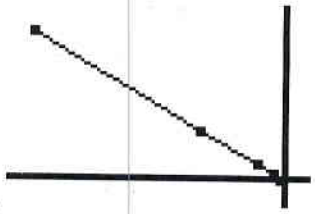
$\lambda = \frac{1.24 \pm \sqrt{1.24^2 - 4(1.03)}}{2} \approx .62 \pm .8035i$

$\sqrt{.62^2 + .8035^2} \approx 1.0149$ rotation w/ very slow repeller

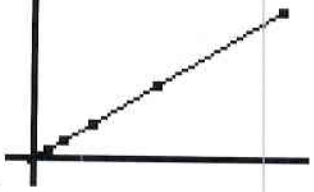
1.a.



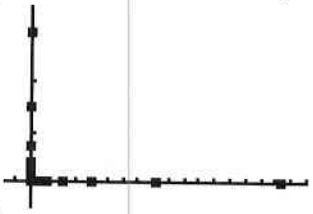
b.



c.



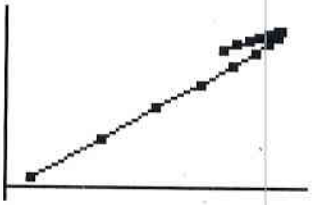
d.



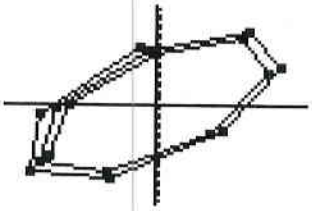
e.

f.

g.



h.



i.