8/29/2024

Symmetry
Transformations of graphs

Symmetry

In relations: x-axis symmetry, y-axis symmetry, origin symmetry
In functions: y-axis = even functions, origin symmetry = odd functions.

Test for x-axis symmetry: replace y in the equation with -y and then see if the equation simplifies to the same thing (is the new point also on the relation?).

Test for y-axis symmetry: replace x in the equation with -x and then see if the equation simplifies to the same equation (is the new point also on the relation?).

For origin symmetry: replace both x and y with -x and -y respectively, and then see if the equation simplifies to the same equation (is the new point also on the relation?).
(You can be origin symmetric and be neither x or y symmetric, or you can be both x and y symmetric and then you must also be origin symmetric).

Equations to test:

$$y = x^2$$
$$x = y^2$$
$$y = x$$
$$x^2 + y^2 = 9$$

Algebraically, test for symmetry of these equations.
x-symmetry:

$$-y = x^2 \to y = -x^2$$

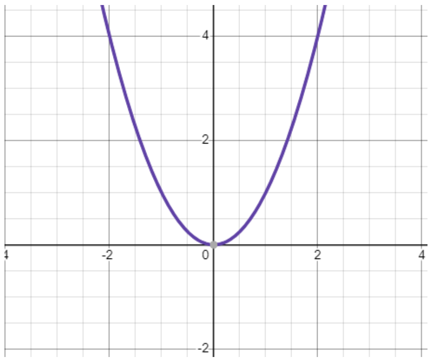
Is that the same equation I started with? No. Not x-symmetric
y-symmetry:

$$y = (-x)^2 = x^2$$

This does simplify to the same equation, so it is y-symmetric
It can’t be origin symmetric:

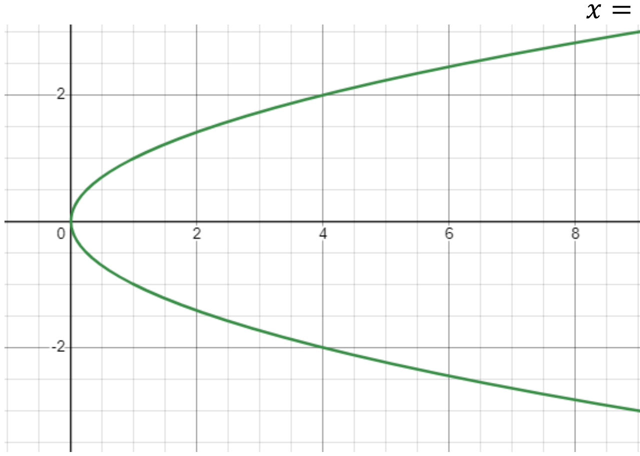
$$-y = (-x)^2 \to -y = x^2 \to y = -x^2$$

Not origin symmetric



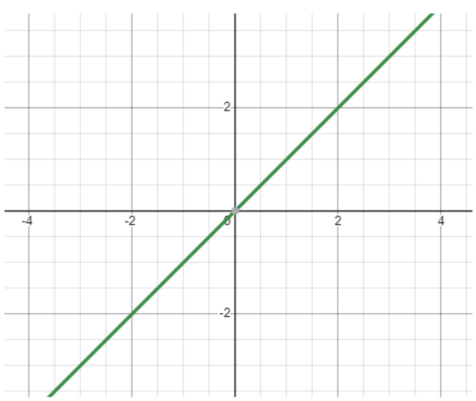
we can see the y-axis symmetry because the right side of the y-axis looks like the left side of the y-axis.

$$x = y^2$$



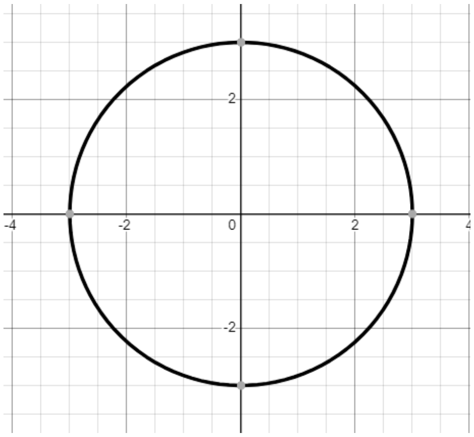
this graph is x-axis symmetric, but it's also not a function.

$$y = x$$



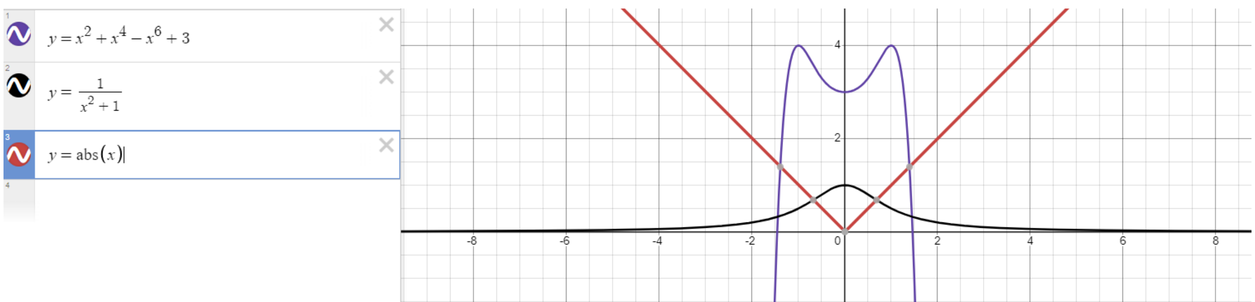
this graph is only origin symmetric

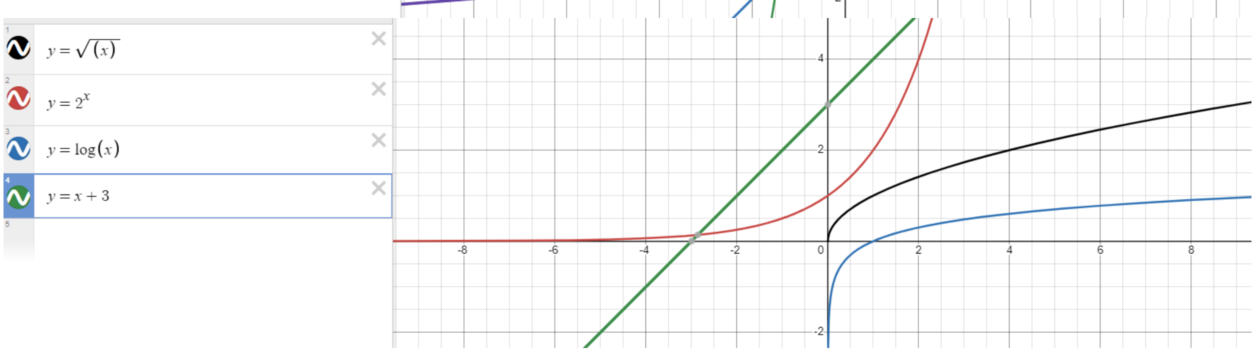
$$x^2 + y^2 = 9$$



This graph is all three symmetries at once.

For functions: you have to be able to solve for y alone.
Even function (like even powered polynomials) are y-symmetric
Odd functions (like odd powered polynomials) are origin symmetric
Mixtures of powers are neither.





Function notation:
Replaces y in the equation with f(x) (read as “f of x”)

$$y = x^2$$
$$f(x) = x^2$$

What is $f(3)$?
What is the value of the function when I put 3 in for x?

$$f(3) = 3^2 = 9$$

$$f(x-1)$$

Replace x in my original equation with x-1 instead:

$$f(x-1) = (x-1)^2$$

Operations on functions

$$f(x) = x^2 + 1, g(x) = \frac{1}{x}$$

$$(f+g)(x) = f(x) + g(x) = x^2 + 1 + \frac{1}{x}$$

$$(f-g)(x) = f(x) - g(x) = x^2 + 1 - \frac{1}{x}$$

$$(fg)(x) = f(x)g(x) = (x^2+1)\left(\frac{1}{x}\right) = \frac{x^2+1}{x}$$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{x^2+1}{\frac{1}{x}} = x(x^2+1) = x^3 + x$$

Difference Quotient

$$\frac{f(x+h) - f(x)}{h}$$

$$f(x) = x^2 + 1$$

$$f(x+h) = (x+h)^2 + 1 = x^2 + 2xh + h^2 + 1$$

$$(x+h)(x+h) = x^2 + xh + xh + h^2 = x^2 + 2xh + h^2$$

$$\frac{\left(x^2 + 2xh + h^2 + 1 - (x^2+1)\right)}{h} = \frac{x^2 + 2xh + h^2 + 1 - x^2 - 1}{h} = \frac{2xh + h^2}{h} = \frac{h(2x+h)}{h} = 2x + h$$

If the algebra is correct, the h in the denominator should always be able to cancel.

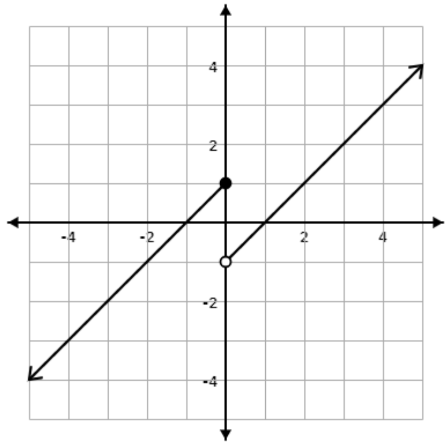
The average cost function is the cost function divided by x

$$\bar{C}(x) = \frac{C(x)}{x}$$

Piecewise functions

$$f(x) = \begin{cases} x+1, x \le 0 \\ x-1, x > 0 \end{cases}$$

https://www.graphfree.com/



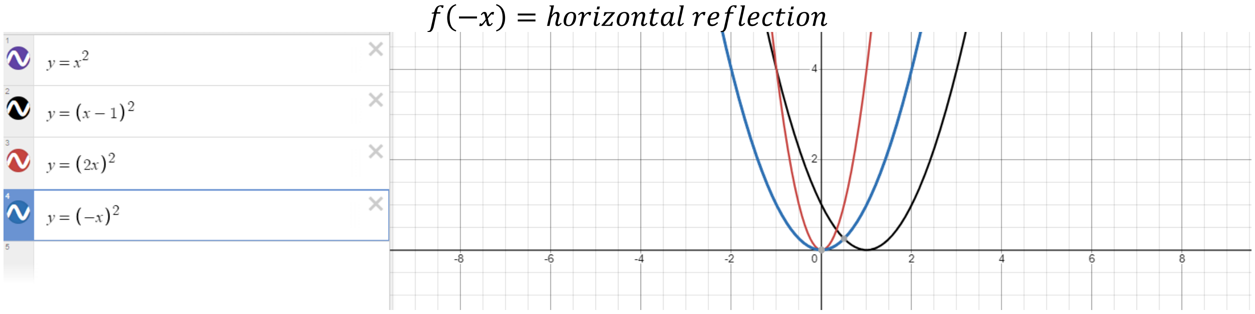
$$f(2) = 2 - 1 = 1$$

Transformations
Vertical Transformations and Horizontal Transformations
Stretch/Compress, Reflect, Shift

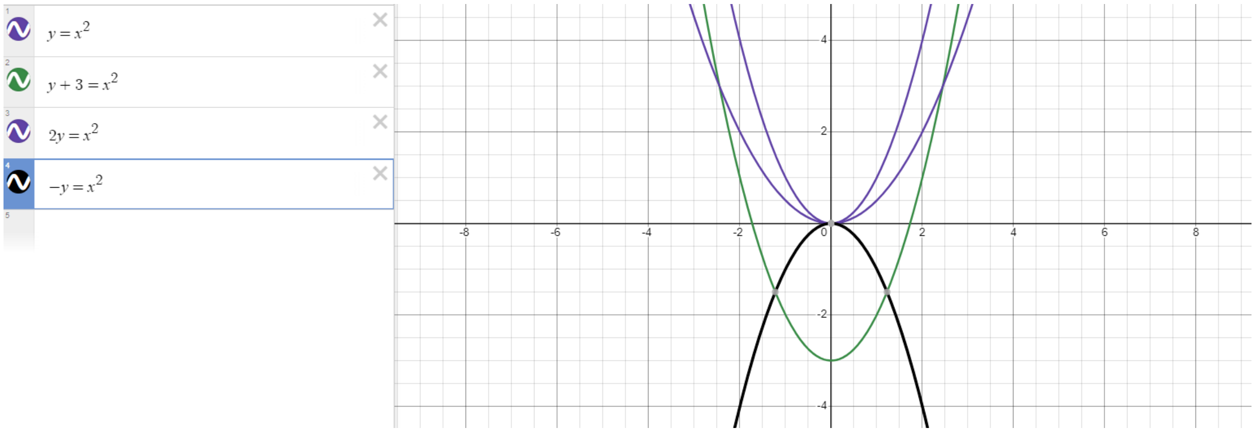
Stretching/Compressing multiplying by a constant
Reflecting multiplies by a negative
Shifting adds or subtracts from the variable

In function notation, any transformation inside the function notation is applied to x and is horizontal transformation, and any transformation applied outside the function notation is applied to y, and is a vertical transformation

$$f(x-1) = horizontal\ shift$$
$$f(2x) = horizontal\ compression$$
$$f(-x) = horizontal\ reflection$$



$$f(x) + 3 = vertical\ shift$$
$$2f(x) = vertical\ stretch$$
$$-f(x) = vertical\ reflection$$



Next time: do another example with square roots

And do them in combinations.
We'll also look at them with piecewise functions and point-to-point