

10/8/2024

Analyzing Rational Functions (continued)

Properties: Domain and range

Vertical asymptotes and holes (factors that cancel)

Horizontal and oblique/slant asymptotes

Intercepts

Sign diagrams

Example.

$$f(x) = \frac{4}{x+2}$$

Domain: $(-\infty, -2) \cup (-2, \infty)$

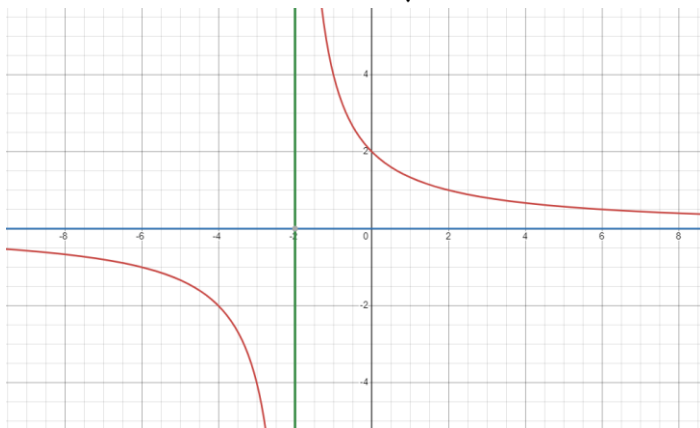
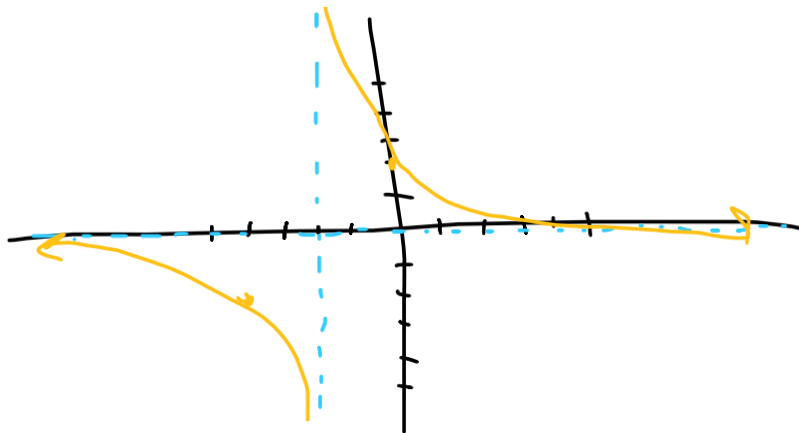
VA: $x = -2$

No holes (no factors can cancel)

HA: $y = 0$

No x-intercepts because the function can never zero

y-intercept: when $x=0$, $y=4/2 = 2$ (0,2)



Example.

$$f(x) = \frac{5x}{6-2x}$$

Domain: $6 - 2x \neq 0, 2x \neq 6, x \neq 3 \rightarrow (-\infty, 3) \cup (3, \infty)$

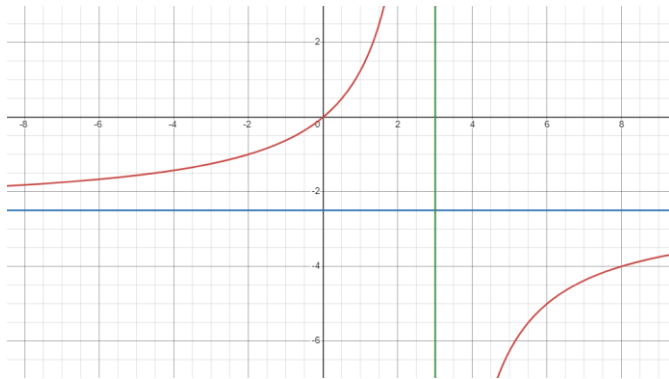
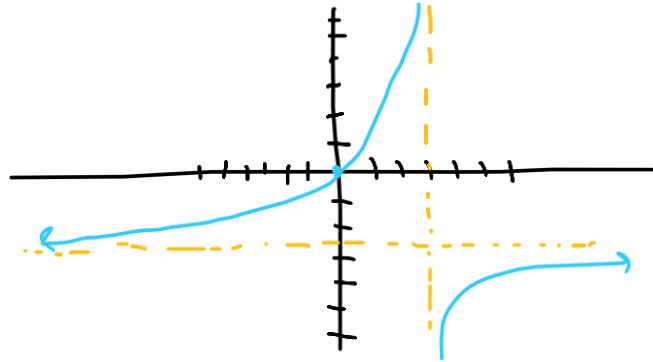
VA: $x=3$

No holes

HA: $\frac{5}{-2} = -\frac{5}{2} = y$

Intercepts:

$$x = 0, y = 0$$



Example.

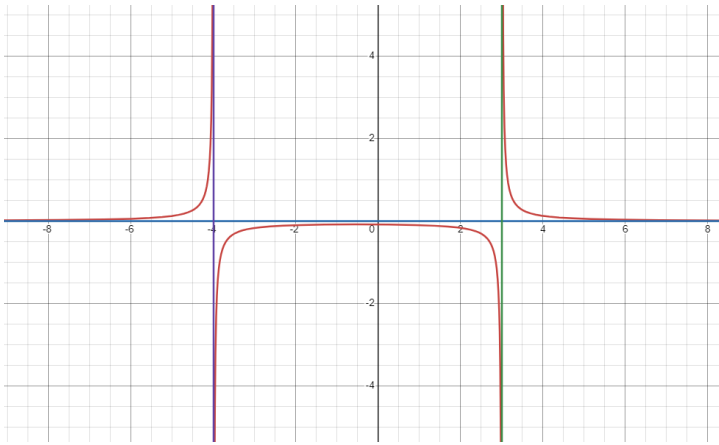
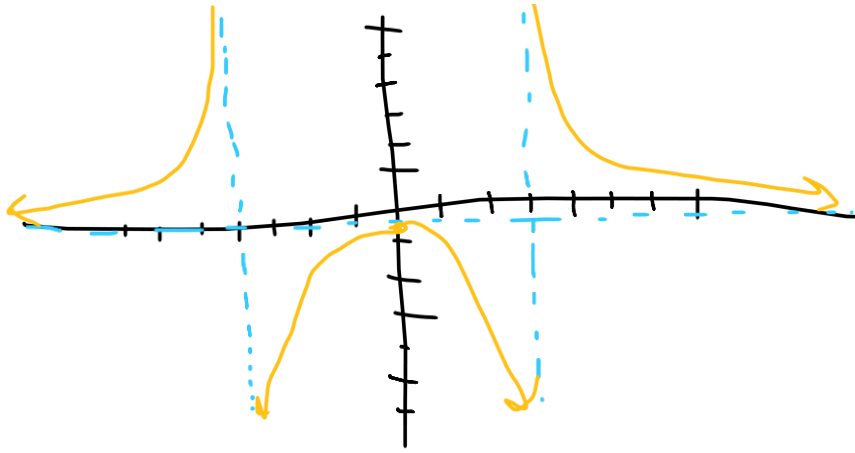
$$f(x) = \frac{1}{x^2 + x - 12}$$

Domain: $x^2 + x - 12 = 0 \rightarrow (x + 4)(x - 3) = 0 \rightarrow x = 3, -4 \dots \rightarrow (-\infty, -4) \cup (-4, 3) \cup (3, \infty)$

VA: $x = 3, x = -4$

HA: $y=0$

Intercepts: no x-intercepts, y-intercept when $x=0, y = \frac{1}{-12}$



Example.

$$f(x) = \frac{4x}{x^2 + 4}$$

Domain: all reals

VA: none, no holes

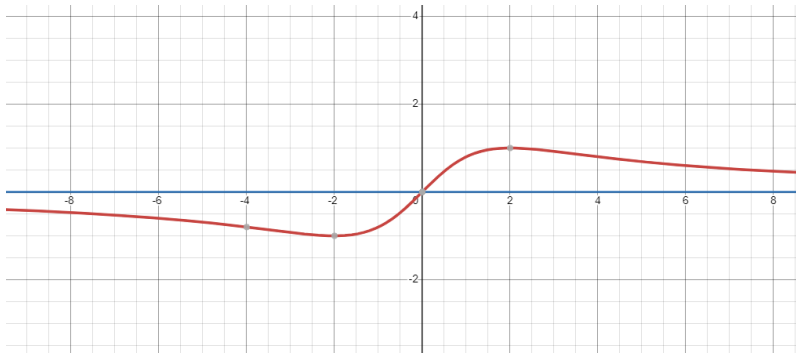
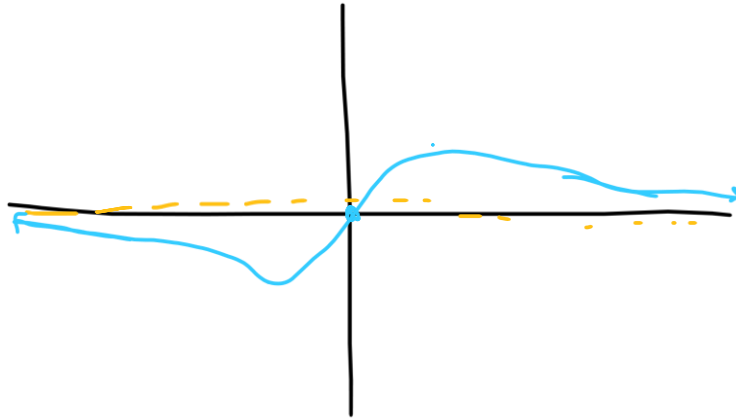
HA: $y = 0$

Intercepts: x-intercept when $x=0$, $(0,0)$ (also y-intercept)

Sign chart

Sign changes when the numerator or the denominator is equal to 0.





Example.

$$f(x) = \frac{x^2 - x}{3 - x} = \frac{x(x - 1)}{3 - x} = -\frac{x(x - 1)}{x - 3}$$

Domain: $x \neq 3, (-\infty, 3) \cup (3, \infty)$

VA: $x = 3$

No holes

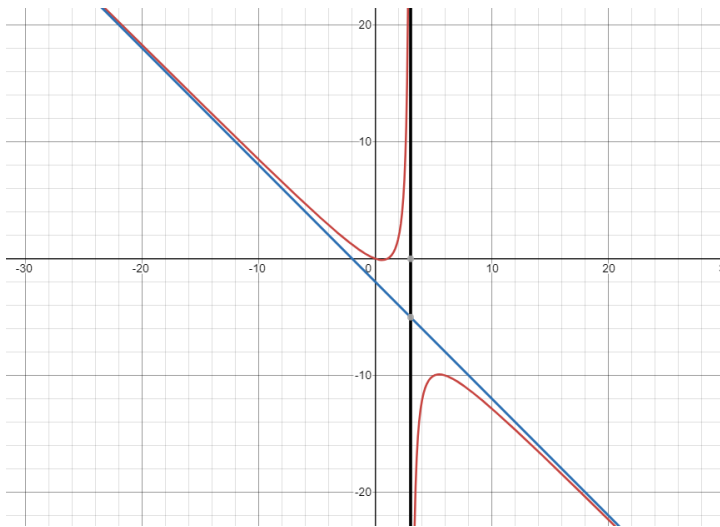
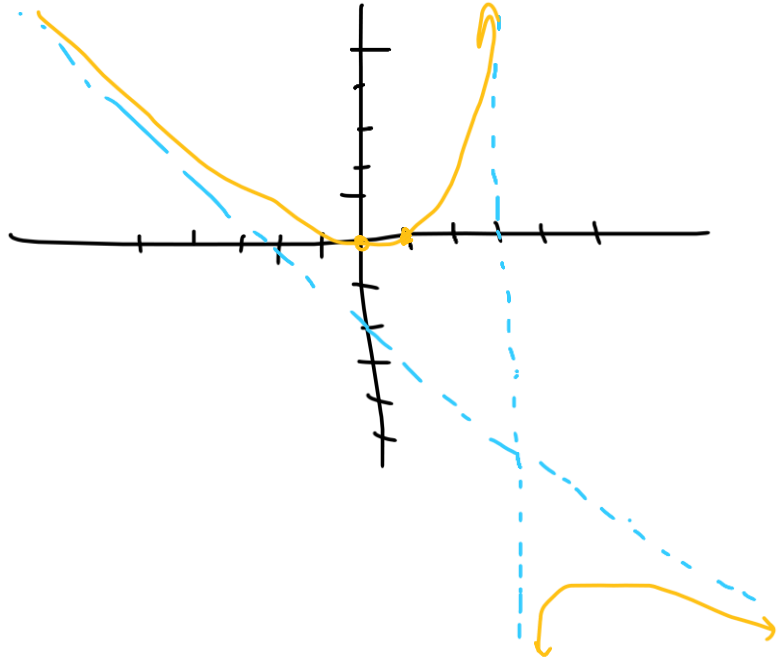
No HA, is a slant asymptote

$$\begin{array}{r|rrr} 3 & 1 & -1 & 0 \\ & & 3 & 6 \\ \hline & 1 & 2 & 6 \end{array}$$

$$-x - 2 - \frac{6}{x - 3}$$

SA: $y = -x - 2$

Intercepts: x-intercepts at $x=0, x=1$



Example.

$$f(x) = \frac{x^3 + 2x^2 + x}{x^2 - x - 2} = \frac{x(x^2 + 2x + 1)}{(x-2)(x+1)} = \frac{x(x+1)^2}{(x-2)(x+1)} = \frac{x(x+1)}{x-2} = \frac{x^2 + x}{x-2}$$

Domain: $(-\infty, -1) \cup (-1, 2) \cup (2, \infty)$

VA: $x = 2$

Hole: $x = -1$ (that's the factor that cancelled) $(-1, 0)$

HA: none

SA: $y = x + 3$

$$2 \sqrt{\begin{array}{ccc|c} 1 & 1 & 0 & \\ & 2 & 6 & \\ \hline 1 & 3 & 6 & \end{array}}$$

$$x + 3 + \frac{6}{x-2}$$

Intercepts: (0,0)

