MTH 161, Homework #3, Fall 2024 Name ______________________________________

Instructions: Show work on a separate sheet of paper and attach to this page. You may check your work with technology not available in class, but you should be able to solve problems and show work without such technology.

1. Simplify the following complex numbers/expressions. Write your solutions in standard form.
   a. $7-(-9+2i)-(-17-i)$
   b. $(2+3i)^2$
   c. $-\frac{6i}{3+2i}$
   d. $5\sqrt{-8}+3\sqrt{-18}$
   e. $(3\sqrt{-5})(-4\sqrt{-12})$
   f. $\frac{1+i}{2+i}+\frac{1-i}{2-i}$

2. Find the solutions, real and complex, of the following functions.
   a. $x^2-6x+10=0$
   b. $3x^2=4x-6$

3. For each of the quadratic functions below, find the vertex and axis of symmetry, any intercepts, and sketch the graph of the function.
   a. $f(x)=3x^2-12x+1$
   b. $f(x)=\frac{5}{4}-\left(x-\frac{1}{2}\right)^2$
   c. $f(x)=-2x^2-12x+3$

4. Find the zeros for each polynomial function, and give the multiplicity for each zero. Describe the end behavior of the polynomial at both $-\infty$ and ∞.
   a. $f(x)=-3\left(x+\frac{1}{2}\right)(x-4)^3$
   b. $f(x)=x^3+7x^2-4x-28$
   c. $f(x)=x^4-x^2$
   d. $f(x)=6x^3-9x-x^5$
   e. $f(x)=x^2(x-1)^3(x+2)$

5. Write a polynomial function that has the following properties.
   a. Zeros at -2, 1, 3 (all multiplicity 1), $f(0)=12$
   b. Zeros at -3, 0 (multiplicity 2), 2 (multiplicity three), $f(1)=-6$
   c. Zeros at -2 (multiplicity 3), -1, $2+3i$, $f(0)=24$
   d. Zeros at -1 (multiplicity 2), 1, $1+i, 2-i$, $f(0)=60$

6. Perform the following divisions. If the factor is linear, perform the division using both long division and synthetic division. Be sure that you get the same answer in both cases. If there is a Remainder, write the result as $Quotient+\frac{Remainder}{Divisor}$.
   a. $(x^2+8x+15)\div(x+5)$
   b. $(12x^2+x-4)\div(3x-2)$
   c. $\frac{x^4+2x^3-4x^2-5x-6}{x^2+x-2}$
   d. $(6x^5-2x^3+4x^2-3x+1)\div(x-2)$

e. $\frac{x^7-128}{x-2}$

7. Use the Remainder Theorem to find the indicated value of the function.
   a. $f(x) = 2x^3 - 11x^2 + 7x - 5, f(4)$
   b. $f(x) = x^4 - 5x^3 + 5x^2 + 5x - 6, f(2)$

8. Use the Rational Zeros Theorem to list all the possible rational zeros of the polynomial. Use Descartes' Rule of Signs to determine the number of possible positive and negative real zeros of the function. Use that information to describe a procedure to checking zeros to reduce the polynomial for factoring.
   a. $f(x) = 3x^4 - 11x^3 - x^2 + 19x + 6$
   b. $f(x) = x^5 - x^4 - 7x^3 + 7x^2 - 12x - 12$
   c. $f(x) = x^3 - 4x^2 + 8x - 5$
   d. $f(x) = 4x^4 - x^3 + 5x^2 - 2x - 6$

9. For each rational function below, find i) the domain of the function, ii) any vertical asymptotes or holes, iii) any horizontal asymptotes or slant asymptotes, iv) any x- or y-intercepts. Sketch the graph.
   a. $f(x) = \frac{3x^2}{(x-5)(x+4)}$
   b. $g(x) = \frac{x+8}{x^2+64}$
   c. $h(x) = \frac{x(x-3)}{x^2-9}$
   d. $r(x) = \frac{-2x+1}{3x+5}$
   e. $q(x) = \frac{x^2-4x+3}{(x+1)^2}$
   f. $s(x) = \frac{x-\frac{1}{x}}{x+\frac{1}{x}}$

10. Find the solution to the polynomial or rational inequality and write the solution in interval notation.
    a. $(x-7)(x+3) \geq 0$
    b. $x^2 \leq 4x - 2$
    c. $x^3 > 9x^2$
    d. $\frac{x}{x-3} < 0$
    e. $\frac{x+1}{x+3} \leq 2$
    f. $\frac{1}{x+1} \geq \frac{2}{x+4}$

11. Solve the variation problems.
    a. y varies directly as x; $y = 65$ when $x = 5$. Find y when $x = 12$.
    b. y varies inversely as x; $y = 12$ when $x = 5$. Find y when $x = 2$.
    c. y varies directly as x and inversely as the square of z; $y = 20$ when $x = 50, z = 5$. Find y when $x = 3, z = 6$.
    d. y varies jointly as a and b, and inversely as the square root of c; $y = 12$ when $a = 3, b = 2, c = 25$. Find y when $a = 5, b = 3, c = 9$.
    e. x varies directly as z and inversely as the difference between y and w.