MTH 264, Quiz #8, Fall 2023 Name KEY

Instructions: Work the problems below as directed. Show all work. Clearly mark your final answers. Use exact values unless the problem specifically directs you to round. Simplify as much as possible. Partial credit is possible, but solutions without work will not receive full credit.

1. Determine whether the infinite series converges or diverges. If it converges, say what it converges to.

   a. $\sum_{n=0}^{\infty}\left(\frac{2}{5}\right)^n$ Converges $r<1$

   $= \frac{1}{1-\frac{2}{5}} = \frac{1}{\frac{3}{5}} = \frac{5}{3}$ by geometric series test

   b. $\sum_{n=1}^{\infty}\frac{6}{n(n+3)}$ telescoping series $\frac{A}{n}+\frac{B}{n+3} \to An+3A+Bn=6$

   $A+B=0$

   $3A=6 \to A=2,\ B=-2$

   $\sum_{n=1}^{\infty} \frac{2}{n}-\frac{2}{n+3}$

   $\frac{2}{1}+\frac{2}{2}+\frac{2}{3}-\left[\lim_{n\to\infty} \frac{2}{n+1}+\frac{2}{n+2}+\frac{2}{n+3}\right]^{\to 0} = 2+1+\frac{2}{3} = 3+\frac{2}{3} = \frac{11}{3}$ Converges

2. Determine whether the infinite series converges or diverges. State which test you used. [Note: you may not use the root or ratio tests.]

   a. $\sum_{n=0}^{\infty}\frac{n+2}{n+1}$ diverges by n-term test limit of sequence $\neq 0$

   b. $\sum_{n=1}^{\infty}\frac{n}{n^4+1}$ $\int \frac{\cancel{x}}{\cancel{x}^4+1}\,dx$ $u=x^2$, $du=2x\,dx$ $\int \frac{1}{2}\frac{1}{u^2+1}\,du = \frac{1}{2}\arctan u \to \frac{1}{2}\arctan x^2\Big|_1^\infty$

   $\lim_{b\to\infty} \frac{1}{2}\arctan b^2 - \frac{1}{2}\arctan 1 = \frac{1}{2}\left(\frac{\pi}{2}\right)-\frac{1}{2}\left(\frac{\pi}{4}\right) = \frac{\pi}{4}-\frac{\pi}{8} = \frac{\pi}{8}$ Converges by integral test

   c. $\sum_{n=1}^{\infty}\frac{1}{n\sqrt{n^2+1}}$ $\lim_{n\to\infty} \frac{1}{n\sqrt{n^2+1}}\cdot\frac{n^2}{1} = \lim_{n\to\infty}\frac{n}{\sqrt{n^2+1}} = 1$

   limit comparison w/ $1/n^2$ (converges by p-series test)

   Converges

   d. $\sum_{n=0}^{\infty}\frac{\cos n\pi}{n+1} = \sum_{n=0}^{\infty}\frac{(-1)^n}{n+1}$ $\lim_{n\to\infty}\frac{1}{n+1}=0$ converges by the alternating series test