MTH 264, Quiz #2, Fall 2023 Name KEY

Instructions: Work the problems below as directed. Show all work. Clearly mark your final answers. Use exact values unless the problem specifically directs you to round. Simplify as much as possible. Partial credit is possible, but solutions without work will not receive full credit.

1. Find the arc length of the function $y = \frac{1}{x+1}$ on the interval [0,1].

$y' = -(x+1)^{-2}$

$$S = \int_0^1 \sqrt{1 + (-(x+1)^{-2})^2}\, dx = \int_0^1 \sqrt{1 + \frac{1}{(x+1)^4}}\, dx \approx 1.13209\ldots$$

2. Find the surface area of the surface generated by revolving the curve $y = \sqrt[3]{x} + 2$ on the interval [1,8] around the y-axis.

$y' = \frac{1}{3}x^{-2/3}$

$$S = 2\pi \int_1^8 x \sqrt{1 + (\tfrac{1}{3}x^{-2/3})^2}\, dx = 2\pi \int_1^8 x \sqrt{1 + \frac{1}{9x^{4/3}}}\, dx$$

$$\approx 2\pi (31.798\ldots) \approx 199.4804797\ldots$$

3. A force of 800 N stretches a spring 70 centimeters on a mechanical device for driving fence posts. Find the work done in stretching the spring the required 70 centimeters.

$F = kx \quad 800 = k(.7) \quad k = \frac{8000}{7}$

$$W = \int_0^{.7} \frac{8000}{7} x\, dx = \frac{4000}{7} x^2 \Big|_0^{.7} = 280 \text{ N·m.}$$

4. Find the value of K that makes $f(x) = Kx^4$ a valid probability distribution on the interval [0.3].

$$\int_0^3 Kx^4\, dx = \frac{K}{5}x^5 \Big|_0^3 = \frac{K}{5}(243) = 1 \rightarrow K = \frac{5}{243}$$