MTH 264, Quiz #13, Fall 2023 Name KEY

Instructions: Work the problems below as directed. Show all work. Clearly mark your final answers. Use exact values unless the problem specifically directs you to round. Simplify as much as possible. Partial credit is possible, but solutions without work will not receive full credit.

1. Solve the differential equations.

   a. $y' = \frac{5x}{y}$

$$\int y\,dy = \int 5x\,dx$$

$$\frac{1}{2}y^2 = \frac{5}{2}x^2 + C$$

$$y^2 = 5x^2 + C$$

   b. $xy + y' = 100x$

$$y' = 100x - xy$$

$$y' = x(100 - y) = -x(y - 100)$$

$$\int \frac{dy}{y-100} = \int -x\,dx$$

$$\ln|y-100| = -\frac{1}{2}x^2 + C$$

$$y - 100 = e^{-\frac{1}{2}x^2 + C} = Ae^{-x^2/2}$$

$$y = Ae^{-x^2/2} + 100$$

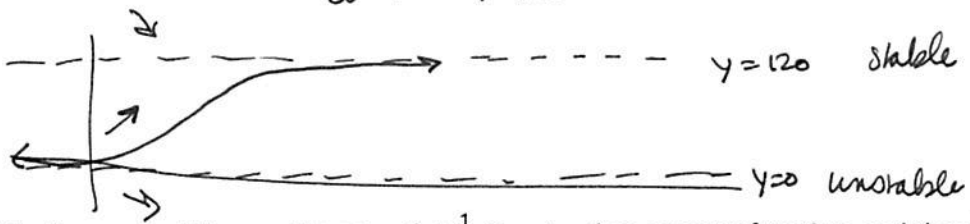
2. Use a direction field to plot the autonomous differential equation $\frac{dy}{dt} = \frac{4y}{5} - \frac{y^2}{150}$ [Hint: factor and plot equilibria.] Determine the properties of each equilibrium and plot the trajectory from the initial condition y=8 when t=0.

See graph

$$\frac{dy}{dt} = \frac{y}{5}\left(4 - \frac{y}{30}\right)$$

$y=0$

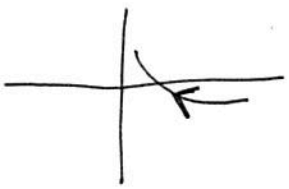
$\frac{y}{30} = 4 \rightarrow y = 120$



3. Write the parametric equations $x = 1 + \frac{1}{t}, y = t - 1$ as a vector function, and then plot the curve. Clearly label the orientation.

$$r(t) = \langle 1 + \tfrac{1}{t},\ t - 1 \rangle \text{ or } \left(1 + \tfrac{1}{t}\right)\hat{i} + (t-1)\hat{j}$$

See attached graph



$$x(t) = 1 + 1/t$$
$$y(t) = t - 1$$