

Instructions: Work the problems below as directed. Show all work. Clearly mark your final answers. Use exact values unless the problem specifically directs you to round. Simplify as much as possible. Partial credit is possible, but solutions without work will not receive full credit.

1. For the differential equations below, determine if they are i) ordinary or partial, ii) linear or non-linear, iii) order.

a.  $\frac{dy}{dx} - xe^y = x$     1) ordinary, 2) nonlinear, 3) first order

b.  $u_x - u_{yy} = x^2$     1) partial, 2) linear, 3) second order

2. Use technology to sketch the direction field  $\frac{dy}{dx} = x^2 - y^2$

See attached graph.

3. Verify that the expression  $y^2 - 2 \ln y = x^2$  is a solution to the differential equation  $\frac{dy}{dx} = \frac{xy}{y^2-1}$ .

$$2y y' - 2 \frac{1}{y} \cdot y' = 2x$$

$$y' \left( 2y - \frac{2}{y} \right) = 2x$$

$$\frac{dy}{dx} = \frac{2x}{2y - \frac{2}{y}} \cdot \frac{y}{y} = \frac{2xy}{2y^2 - 2} = \frac{xy}{y^2 - 1} \quad \checkmark$$

4. Use Euler's method to approximate the value of the solution to  $y' = \frac{1}{2}x(3-y)$  at  $y(2)$ , starting at  $y(0) = 1$ , using  $n = 5$  steps.

$$\Delta x = \frac{2-0}{5} = \frac{2}{5} = 0.4$$

$$m_1 = \frac{1}{2}(0)(3-1) = 0 \quad (0.4, 1)$$

$$y_1 = 0(\Delta x) + y_0 = 1 \quad (0.8, 1.16)$$

$$m_2 = \frac{1}{2}(0.4)(3-1) = 0.2 \quad (1.2, 1.4544)$$

$$y_2 = 0.2(\Delta x) + y_1 = 1.16 \quad (1.6, 1.825344)$$

$$m_3 = \frac{1}{2}(0.8)(3-1.16) = 0.736 \quad (2, 2.20123392)$$

$$y_3 = 0.736(\Delta x) + y_2 = 1.16$$

$$m_3 = \frac{1}{2}(0.8)(3-1.16) = 0.736$$

$$y_3 = 0.736(0.4) + 1.16 = 1.4544$$

$$m_4 = \frac{1}{2}(1.2)(3-1.4544) = 0.92736$$

$$y_4 = 0.92736(0.4) + 1.4544 = 1.825344$$

$$m_5 = \frac{1}{2}(1.6)(3-1.825344) = 0.9397248$$

$$y_5 = 0.9397248(0.4) + 1.825344 = 2.20123392$$

