MTH 267, Quiz #9, Fall 2022 Name KEY

Instructions: Show all work. Answers without work required to obtain the solution will not receive full credit. Some questions may contain multiple parts: be sure to answer all of them. Give exact answers unless specifically asked to estimate.

1. Find the Laplace transform of $f(t) = t$ **using the definition**. Recall $L(f) = \int_0^\infty e^{-st} f(t)dt$.

$$\int_0^\infty e^{-st} \cdot t \, dt \qquad u = t \quad dv = e^{-st}dt \qquad du = dt \quad v = -\frac{1}{s}e^{-st}$$

$$-\frac{1}{s}te^{st} + \int_0^\infty +\frac{1}{s}e^{-st}dt = -\frac{1}{s}te^{-st} - \frac{1}{s^2}e^{-st}\Big|_0^\infty = 0 - 0 + 0 + \frac{1}{s^2}$$

$$= \frac{1}{s^2}$$

2. Use the table to find the inverse Laplace transform of $F(s) = \frac{1-2s}{s^2+4s+5}$.

$$\cancel{\frac{1-2s}{(s+4)}} = \frac{1-2s}{(s^2+4s+4)+1} = \frac{1-2s}{(s+2)^2+1} = \frac{5}{(s+2)^2+1} - \frac{2(s+2)}{(s+2)^2+1}$$

$1 - 2(s+2) + 4$

$1 - 2s - 4 + 4$

$5 - 2(s+2)$

$a = -2$

$b = 1$

$$\boxed{f(t) = 5e^{-2t}\sin t - 2e^{-2t}\cos t}$$