MTH 267, Quiz #6, Fall 2022 Name KEY

Instructions: Show all work. Answers without work required to obtain the solution will not receive full credit. Some questions may contain multiple parts: be sure to answer all of them. Give exact answers unless specifically asked to estimate.

1. Use undetermined coefficients to find the general solution to $y'' + 4y = 5\sin(t)$.

$r^2+4=0 \quad y = c_1\cos 2t + c_2 \sin 2t$

$r=\pm 2 \quad Y(t) = A\sin t + B\cos t$ - Ansatz

$Y' = A\cos t - B\sin t$

$Y'' = -A\sin t - B\cos t$

$-A\sin t - B\cos t + 4A\sin t + 4B\cos t = 5\sin t$

$3A\sin t + 3B\cos t = 5\sin t \qquad B=0$

$3A\sin t = 5\sin t$

$3A = 5 \Rightarrow A = 5/3$

$y(t) = c_1\cos 2t + c_2\sin 2t + 5/3\sin t$

2. Use variation of parameters to find the general solution to $y'' + 4y = 5\csc(2t)$.

$y_1 = \cos 2t, \quad y_2 = \sin 2t$

$W = \begin{vmatrix} \cos 2t & \sin 2t \\ -2\sin 2t & 2\cos 2t \end{vmatrix} = 2\cos^2 2t + 2\sin^2 2t = 2$

$Y_p = -\cos 2t \int \frac{\sin 2t \cdot 5\csc 2t}{2}\,dt + \sin 2t \int \frac{\cos 2t \cdot 5\csc 2t}{2}\,dt =$

$-\cos 2t \cdot \frac{5}{2}\int dt + \frac{5}{2}\sin 2t \int \frac{\cos 2t}{\sin 2t}\,dt = -\frac{5}{2}\cos 2t \cdot t + \frac{5}{4}\sin 2t \ln|\sin 2t|$

$u = \sin 2t$

$du = 2\cos 2t\,dt$

$\frac{1}{2}\int \frac{1}{u}\,du$

$y(t) = c_1\cos 2t + c_2\sin 2t - \frac{5}{2}t\cos 2t + \frac{5}{4}\sin 2t \cdot \ln|\sin 2t|$