MTH 267, Quiz #5, Fall 2022 Name KEY

Instructions: Show all work. Answers without work required to obtain the solution will not receive full credit. Some questions may contain multiple parts: be sure to answer all of them. Give exact answers unless specifically asked to estimate.

1. Solve the differential equations below.

a. $y'' + 4y' + 12y = 0, y(0) = 3, y'(0) = 0$

$r^2 + 4r + 12 = 0$

$(r \quad)(r \quad)$

$r = \frac{-4 \pm \sqrt{16 - 4(12)}}{2} = \frac{-4 \pm \sqrt{-32}}{2}$

$= \frac{-4 \pm 4\sqrt{2}i}{2} = -2 \pm 2\sqrt{2}i$

$y = c_1 e^{-2x}\cos(2\sqrt{2}x) + c_2 e^{-2x}\sin(2\sqrt{2}x)$ → $\boxed{y = 3e^{-2x}\cos(2\sqrt{2}x) + \frac{3}{\sqrt{2}} e^{-2x}\sin(2\sqrt{2}x)}$

$3 = c_1(1)(1) + c_2(1)(0)$ → $c_1 = 3$

$y' = 3(-2)e^{-2x}\cos(2\sqrt{2}x) + -3(2\sqrt{2})e^{-2x}\sin(2\sqrt{2}x) + c_2(-2)e^{-2x}\sin(2\sqrt{2}x) + c_2 e^{-2x}\cos(2\sqrt{2}x) \cdot 2\sqrt{2}$

$0 = -6(1)(1) - 6\sqrt{2}(1)(0) + c_2(-2)(1)(0) + c_2 2\sqrt{2}(1)(1)$

$6 = 2\sqrt{2}c_2$ → $\frac{3}{\sqrt{2}} = c_2$

b. $y'' - 16y = 0$. Write the solution to this problem as hyberbolic trig functions.

$y = c_1\cosh 4x + c_2 \sinh 4x$

$r^2 - 16 = 0$

$r = \pm 4$

$y = c_1 e^{-4x} + c_2 e^{4x}$

c. $t^2y'' - 6ty' + 12y = 0, y(1) = 2, y(2) = 6$

$y = t^r \quad y' = rt^{r-1} \quad y'' = r(r-1)t^{r-2}$

$t^2(r)(r-1)t^{r-2} - 6tr(t^{r-1}) + 12t^r = 0$

$t^r(r^2 - r) - 6r(t^r) + 12t^r = 0$

$t^r[r^2 - r - 6r + 12] = 0$

$r^2 - 7r + 12 = 0$

$(r-3)(r-4) = 0 \quad r = 3, 4$

$y = c_1 t^3 + c_2 t^4$

$2 = c_1(1)^3 + c_2(1)^4$

$2 = c_1 + c_2$

$6 = c_1(2)^3 + c_2(2)^4$

$6 = 8c_1 + 16c_2$

$c_1 + c_2 = 2 \quad \times -8$ → $-8c_1 - 8c_2 = -16$

$8c_1 + 16c_2 = 6$

$8c_2 = -10$

$c_2 = -10/8 = -5/4$

$c_1 - 5/4 = 2$

$c_1 = \frac{8}{4} + \frac{5}{4} = 13/4$

$y = 13/4 t^3 - 5/4 t^4$