MTH 267, Quiz #4, Fall 2022 Name KEY

Instructions: Show all work. Answers without work required to obtain the solution will not receive full credit. Some questions may contain multiple parts: be sure to answer all of them. Give exact answers unless specifically asked to estimate.

1. Solve the exact equation
$\left(3x^2y - 4xy^2 - e^x \sin y - \frac{1}{x+1}\right)dx + (x^3 - 4x^2y - e^x \cos y + 1)dy = 0$, y(0)=0.

$\int 3x^2y - 4xy^2 - e^x \sin y - \frac{1}{x+1}\,dx = x^3y - 2x^2y^2 - e^x \sin y - \ln|x+1| + g(y)$

$\int x^3 - 4x^2y - e^x \cos y + 1\,dy = x^3y - 2x^2y^2 - e^x \sin y + y + h(x)$

$\varphi(x,y) = x^3y - 2x^2y^2 - e^x \sin y + \ln|x+1| + y + K$

2. A tank has pure water flowing into it at 10 L/min. The contents of the tank are kept thoroughly mixed, and the contents flow out at 10 L/min. Initially, the tank contains 10 kg of salt in 100 L of water. How much salt will there be in the tank after 30 minutes?

$A(0) = 10$

$\frac{dA}{dt} = Rate_{in} - Rate_{out}$

$Rate_{in} = \frac{0\,kg}{L} \cdot \frac{10\,L}{min} = 0 \qquad Rate_{out} = \frac{A\,(kg)}{100\,L} \cdot \frac{10\,L}{min} = \frac{A}{10}$

$\frac{dA}{dt} = -\frac{A}{10} \rightarrow \int \frac{dA}{A} = \int -\frac{1}{10}\,dt = \ln A = -\frac{1}{10}t + C$

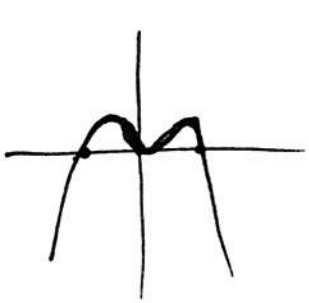
$A = A_0 e^{-\frac{1}{10}t}$

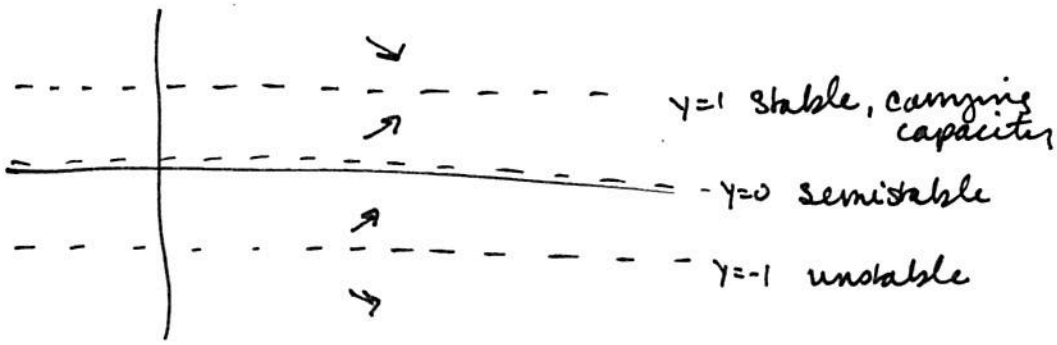
$A = 10e^{-\frac{1}{10}t}$

$A(30) = 0.049787\,kg$

3. Draw the phase plane for the ODE $\frac{dy}{dt} = y^2(1-y^2)$ and use that to characterize each solution as i) stable, unstable or semi-stable; ii) any solution for which $y > 0$ as a threshold, carrying capacity or neither.

$y^2(1-y)(1+y) = 0 \qquad y = 0,\ y = 1,\ y = -1$





4. Use Runge-Kutta to find $y(1)$ for the differential equation $\frac{dy}{dt} = y(y - 2t)$, $y(0) = -2$. Use $\Delta t = 0.1$. Verify one step of your calculation by hand, and then complete the remaining steps with technology (such as Excel).

$n=0$ $k_{n1} = -2(-2 - 2(0)) = 4$

$y_{01} = 4(0.05) + -2 = -1.8$

$k_{n2} = -1.8(-1.8 - 2(0.05)) = 3.42$

$y_{02} = 3.42(0.05) - 1.8 = -1.829$

$k_{n3} = -1.829(-1.829 - 2(0.05)) = 3.528141$

$y_{03} = -2 + 3.528(0.05) = -1.647186$

$k_{n4} = -1.64(-1.64 - 2(0.1)) = 3.042659$

$y_{04} = \eta_1 = -2 + \frac{0.1}{6}(4 + 2(3.42) + 2(3.528141) + 3.042659)$

$= -1.65102$

See Excel