

**Instructions:** Show all work. Answers without work required to obtain the solution will not receive full credit. Some questions may contain multiple parts: be sure to answer all of them. Give exact answers unless specifically asked to estimate.

1. Solve the exact equation

$$\left(3x^2y - 4xy^2 - e^x \sin y - \frac{1}{x+1}\right) dx + (x^3 - 4x^2y - e^x \cos y + 1)dy = 0, y(0)=0.$$

2. A tank has pure water flowing into it at 10 L/min. The contents of the tank are kept thoroughly mixed, and the contents flow out at 10 L/min. Initially, the tank contains 10 kg of salt in 100 L of water. How much salt will there be in the tank after 30 minutes?

3. Draw the phase plane for the ODE  $\frac{dy}{dt} = y^2(1 - y^2)$  and use that to characterize each solution as i) stable, unstable or semi-stable; ii) any solution for which  $y > 0$  as a threshold, carrying capacity or neither.

4. Use Runge-Kutta to find  $y(1)$  for the differential equation  $\frac{dy}{dt} = y(y - 2t)$ ,  $y(0) = -2$ . Use  $\Delta t = 0.1$ . Verify one step of your calculation by hand, and then complete the remaining steps with technology (such as Excel).